Deciding Under Uncertainty

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Introduction

It is difficult to decide what to do, if one is a consumer or businessperson, when one is faced with uncertainty. In this set of notes, we shall look at some concepts which help a person decide what to do when faced with choices that have many possible outcomes, some better than others. In addition, we shall discuss how we can model the different tolerances that individuals have regarding exposure to risk, and we shall see why risk averse individuals are willing to buy insurance.

Let's begin by developing a fake example of a caddy trying to decide for whom to caddy; we shall develop some concepts which will help in her choice.

For whom should Zelda caddy?

Zelda makes her living by caddying on the PGA tour. This week she has the option of caddying for one of two players: Tiger or Sergio. Both are nice guys, but Zelda is more interested in how much money she'll make. Here's what she knows:

About Tiger:

--Tiger pays $2000 when he shoots lower than 69; he pays $1500 when he shoots 69 or higher.

--80% of the time Tiger shoots lower than 69; the rest of the time (20%) he shoots 69 or higher.

About Sergio:

--Sergio pays $3000 when he shoots lower than 69; he pays $200 when he shoots 69 or higher.

--70% of the time Sergio shoots lower than 69; the rest of the time (30%) he shoots 69 or higher.

Dang! What a tough choice! Sergio pays more when he plays well, but he pays a lot less when he plays badly. Tiger's payout also varies depending upon how well he plays, but not as much as Sergio's payout varies.

Probability:

Let's help Zelda decide. First, a review of *probability* and *expected value*. The probability that something will occur (in the future) is expressed as a number, which can range in value from 0 to 1. If an event has a probability of 0 then it will not occur under any circumstance. If an event has a probability of 1 then it will definitely occur under all circumstances.

Most future possible outcomes have a probability lying between 0 and 1. For example, the probability that when you flip a coin it will come up "heads" is ½. Another example: The probability of drawing a queen out of a deck of cards is 4/52.

Zelda's pay and its probability:

--Zelda knows the probability that she will get paid $2000 if she caddies for Tiger: 80%, or .80; she also knows the probability that she will get paid $1500 if she caddies for Tiger: 20%, or .20.

--Zelda knows the probability that she will get paid $3000 if she caddies for Sergio: 70%, or .70; she also knows the probability that she will get paid $200 if she caddies for Sergio: 30%, or .30.

Zelda can do all sorts of things with this probability information. For one thing, she can calculate how much money she will make, *on average*, if she caddies for Tiger or Sergio.

Expected value:

Expected value is simply a weighted average of all possible outcomes of a choice. Each of the possible outcomes is weighed by the probability that it will occur.

Example: You throw a six-sided die. What is the expected value of the throw? Well, you will throw either a 1, 2, 3, 4, 5, or 6. Each has probability 1/6 that it will occur. So the expected value is:

$$E(\text{throw}) = 1/6(1) + 1/6(2) + 1/6(3) + 1/6(4) + 1/6(5) + 1/6(6)$$
$$= 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6$$
$$= 3.5$$

Now let's have Zelda the expected value of caddying for Tiger, and the expected value of caddying for Sergio:

$$E(\text{Tiger}) = .80(2000) + .20(1500) = \$1900$$

$$E(\text{Sergio}) = .70(3000) + .30(200) = \$2160$$

So Zelda should caddy for Sergio, right? Not necessarily! Sergio pays more, on average, but his pay is riskier than Tiger's. Oh, if only we had a measure of the **risk** of the pay from each golfer. We do!

Standard deviation:

One measure of the risk of a choice is its *standard deviation*. This is a measure of how dispersed all of the possible outcomes of a choice are around the expected value (average) of the choice. The larger the standard deviation, the riskier is the choice.

Calculating the standard deviation of a choice is not difficult, but there are a few steps involved.

Calculating standard deviation:

Step 1: Subtract each possible outcome from the expected value.

Step 2: Square each result from step 1

Step 3: Multiply each result from step 2 by the probability of each outcome.

Step 4: Sum all of the results of step 3.

Step 5: Take the square root of step 4.

Let's calculate the standard deviation—the risk of the payout—involved with caddying for **Tiger**

Step 1: 2000 – 1900 = 100
1500 – 1900 = -400

Step 2: $100^2 = 10{,}000$
$-400^2 = 160{,}000$

Step 3: .80(10,000) = 8,000
.20(160,000) = 32,000

Step 4: 8,000 + 32,000 = 40,000

Step 5: standard deviation = $\sqrt{40{,}000}$ = $200

Now let's calculate the standard deviation—the risk of the payout—involved with caddying for **Sergio**

Step 1: 3000 – 2160 = 840
200 – 2160 = -1960

Step 2: $840^2 = 705{,}600$
$-1960^2 = 3{,}841{,}600$

Step 3: .70(705,600) = 493,920
.30(3,841,600) = 1,152,480

Step 4: 493,920 + 1,152,480 = 1,646,400

Step 5: standard deviation = $\sqrt{1{,}646{,}400}$ = $1283.12

So it's much riskier to caddy for Sergio. But he pays more on average then Tiger. Oh, how to decide! We need more information about Zelda. How much does she care about the risk of a payout?

Individuals and Risk

We can put a person into one of three categories, depending upon her tolerance for risk:

Risk Averse: A person is risk averse if she prefers a 100% certain choice over an uncertain choice with the same expected value.

Risk Neutral: A person is risk neutral if she is indifferent between a 100% certain choice to an uncertain choice with the same expected value.

Risk loving: A person is risk loving if she prefers an uncertain choice over a 100% certain choice with the same expected value.

Example: Monty Hall says: "You can have $100 for sure, or you can choose door #1 or door #2. Behind one of the doors is $200. Behind the other door is $0."

--If you say "Monty, I'll take the $100," then you are risk averse.

--If you say "Do whatever the heck you want, Monty—I'll take the $100 or I will choose between the two doors." Then you are risk neutral.

--If you say "Forget the hundred bucks, dude! I'll pick one of the two doors." Then you are risk loving.

Zelda's choice:

Now, you should be able to see the following

--If Zelda is risk neutral or risk loving, then she should definitely caddy for Sergio.

(Why? If Zelda is risk loving, then Sergio is a better job on two counts: his expected payout is larger and his risk, which Zelda loves, is higher. If Zelda is risk neutral, then the risk is irrelevant, and Zelda would only take into account the expected value, which is higher for Sergio.)

--If Zelda is risk averse, then maybe she should caddy for Tiger, depending upon how risk averse she really is.

(Why? If she is only a little risk averse, then Sergio may still be the better choice, since his payout has a higher expected value. If she is a lot risk averse, then Tiger may be the better choice, since his standard deviation is much smaller.)

Risk and Expected Utility:

Zelda gains utility (indirectly) when she is paid, since she can use the income to buy items which give her utility. Economists can model an individual's happiness using a utility function in which the person's total utility depends upon how much income the person receives. (These are sometimes called *indirect* utility functions, since income is only an indirect measure of happiness.) Here are three examples of utility functions in which utility, U, depends on income, I:

$U = I^{1}$ $U = I^{5}$ $U = I^{.2}$

(Notice that in the last utility function, we have I raised to the ".”2)

--A note about the exponent that "I" is raised in the utility functions above:
Believe it or not, if the exponent is less than 1, then the person is risk averse. If the exponent is exactly 1, then the person is risk neutral. If the exponent is greater than 1, then the person is risk loving.

Don't believe me? Hah! Let's demonstrate, using the concept of expected utility.

<u>Expected utility:</u>
Suppose we don't know what will happen when we make a choice. For example, when we buy a lotto ticket, we don't know whether we will lose $1 or win the jackpot. In these cases (as with Zelda's choice above), we don't know how much utility we will get from our choice. But we can calculate a weighted average of the utilities from all of the possible outcomes of the choice. If we do this, we are calculating **expected utility**.

Let's use Zelda as an example. We must give her a utility function. I shall give her the utility function $U = I^{.2}$ (that's "I" raised to the two-tenths power.) Because the exponent in the utility function is less than 1, I have made Zelda risk averse.

Suppose she caddies for **Tiger**:

80% of the time she well get $2000, resulting in utility $U = 2000^{.2}$ = **4.573051**
20% of the time she will get $1500, resulting in utility $U = 1500^{.2}$ = **4.31736**

Hence Zelda's expected utility is .8(**4.573051)** + .2(**4.31736) = 4.521912**

Suppose she caddies for **Sergio**:

70% of the time she well get $3000, resulting in utility $U = 3000^{.2}$ = **4.959344**
30% of the time she will get $200, resulting in utility $U = 200^{.2}$ = **2.8854**

Hence Zelda's expected utility is .7(**4.959344)** + .3(**2.8854) = 4.337161**

The choice with the higher expected utility is the superior choice! Zelda should caddy for Tiger!

A utility-maximizing individual should choose the option with the highest expected utility. This choice maximizes the individual's chance of maximum happiness, given his/her tolerances for risk.

<u>A New Choice for Zelda: Tiger vs. Jack</u>
Now, let's demonstrate that Zelda really is risk averse. Let's give her a new choice: Caddy for Tiger, or caddy for Jack. Jack always pays $1900.

Note the difference between Tiger and Jack:

--Tiger pays \$1900 on average, but the payout is riskier; sometimes it is higher than \$1900 and sometimes it is lower (\$2000 with 80% probability and \$1500 with 20% probability).

--Jack always pays \$1900. There is no risk to caddying for Jack.

A risk averse person should prefer to caddy for Jack. Does Zelda prefer to caddy for Jack? Let's compare her expected utility under the two options:

1) Caddy for Tiger. Expected utility = **4.521912** (We calculated this on p. 5.)

2) Caddy for Jack. 100% of the time she will get \$1900, resulting in utility

$U = 1900^{.2}$ = **4.526377**. Since this is the only possible outcome, this number is also Zelda's expected utility if she caddies for Jack

Jack is the better choice for Zelda. This demonstrates that Zelda is risk averse.

> Moral: When "I" is raised to an exponent less than 1 in an individual's utility function, then the individual is risk averse.

Now let me demonstrate that when the exponent is greater than one that the individual is risk loving.

<u>The Evil Twin of Zelda</u>

Example 2: In an alternate universe, the Evil Twin of Zelda has the same Tiger vs. Jack option: caddy for Jack and make a sure \$1900, or caddy for Tiger and make \$2000 with 80% probability and \$1500 with 20% probability. The Evil Twin of Zelda has a risk-loving utility function:

$$\mathbf{U = I^{1.5}}$$

Suppose Evil Twin of Zelda caddies for **Tiger**:

80% of the time she well get \$2000, resulting in utility $U = 2000^{1.5}$ = **89,443**
20% of the time she will get \$1500, resulting in utility $U = 1500^{1.5}$ = **58,095**

Hence The Evil Twin of Zelda's expected utility is .8(**89,443)** + .2(**58,095) = 83173**

Suppose Evil Twin of Zelda caddies for Jack. 100% of the time she will get \$1900, resulting in utility

$U = 1900^{1.5}$ = **82819** . Since this is the only possible outcome, this number is also The Evil Twin of Zelda's expected utility if she caddies for Jack

You see how The Evil Twin of Zelda prefers the riskier choice, Tiger!!!

Moral: When "I" is raised to an exponent greater than 1 in an individual's utility function, then the individual is risk loving.

Risk Aversion and Insurance

Since a risk averse person prefers a 100% certain payment of income to a risky payment with the same expected value, a risk averse person is willing to pay to avoid risk; that is, she is willing to buy *insurance*.

But how much is she willing to pay? Let's examine this issue.

Example 3:

Here's 3 pieces of information about Mario:

1) Mario has utility function $\mathbf{U = I^{.6}}$ (He is risk averse)

2) The only job income-producing job that Mario can get pays $10000 per year 20% of the time and $50000 per year 80% of the time.

(Expected value of yearly income = .2(10000) + .8(50000) = $42000)

3) An insurance agent approaches Mario and says: "Hey Mario, dude! You don't like risk. You would prefer $42,000 yearly with 100% certainty over the job that you have now. Well, here's the deal:"

"If you give me your salary every year, then I will give you $42,000 per year with 100% certainty, *minus an insurance premium*. So what is the maximum insurance premium that you will pay me for this deal? Eh? Speak up, sonny boy!"

Here's Mario's answer:

Well, with my current job, I receive expected utility of

A) Expected utility of current job =

$.2(10000^{.6}) + .8(50000^{.6}) = 578.0408929$ utils

B) How much income would I need to receive with 100% certainty to receive this same level of utility?

$I^{.6} = 578.0408929$

$(I^{.6})^{(1/.6)} = 578.0408929^{(1/.6)}$

$I = \$40111.07$

C) Now, here's my logic: I will be just as well off if I receive $40,111.07 with 100% certainty as I am with my current job. You offer to pay me $42,000 with certainty, if I pay you an insurance premium. So the most I will pay you for an insurance premium is

$42,000 - $40,111.07 = $**1888.928**

The idea, restated, is this.

If the insurer pays Mario $42,000 per year every year all of the time, then subtracts an insurance premium of $1888.928, then this will leave Mario with spendable income of $40,111.07 per year every year, giving him utility of 578.0408929 utils every year. This level of utility is exactly the same as the expected utility from his current job. Hence Mario will at most pay $1888.928 to take the insurer's offer. (If Mario can somehow get the deal for less than $1888.928, then he is better off taking the insurance than he is turning it down. If he must pay more than $1888.928, then he should turn down the insurance offer.)

And now, a note about risky assets.

Clearly, risk is important to those investing their wealth in financial assets. High returns are good. Risk is bad. Unfortunately, higher returns are usually associated with higher risk.

Diversifying one's ownership of assets reduces risk, but also reduces the probability of really high returns.

In corporate finance class and investment theory class, you will cover the demand for risky assets in excruciating detail.