Study Question – Pricing, Question 2

A firm with market power segments its buyers into two groups:
Elderly (e) and Young (y)

Elderly demand is: $P_e = 1000 - 4Q_e$
Young demand is $P_y = 1600 - Q_y$

The firm has costs according to this cost function:

$C = 100 + Q^2$ (note also: $Q = Q_e + Q_y$)

find the profit-maximizing decisions on output and prices.

We must follow the rule $MR_e = MR_y = MC$

The marginal cost comes from differetiating the cost function, C, with respect to Q

$MC = 2Q$

To get marginal revenue for the two groups, we first convert the demand equations into total revenue equations:

$TR_e = P_eQ_e = (1000 – 4Q_e)Q_e = 1000Q_e – 4Q_e^2$
$TR_y = P_yQ_y = (1600 –Q_y)Q_y = 1600Q_y –Q_y^2$

Next, we differentiate each equation with respect to Q:

$MR_e = 1000 – 8Q_e$ $MR_y = 1600 – 2Q_y$

Now, to find the level of Q_e and Q_y that satisfy the rule **$MR_e = MR_y = MC$,** we must "horizontally sum" the marginal revenue equations, and set the sum equal to MC. Here's one procedure to horizontally sum the MR equations:

Express both MR equations with Q on the left hand side:

$Q_e = 125 - .125MR_e$ $Q_y = 800 - .5MR_y$

Now add the two equations:

$(Q_e + Q_y) = 125 + 800 - .125MR_e - .5MR_y$

$Q = 925 - .625MR$

Now let's get MR back on the left hand side:
$MR = (925/.625) – (Q/.625)$ or **$MR = 1480 – 1.6Q$**

Now, let's set the combined MR equation equal to MC, and solve for Q:

$MR = MC$
$1480 – 1.6Q = 2Q$ $3.6Q = 1480$ **$Q = 411.111$**

Hence total production, Qe + Qy, is 411.111.

But wait—how is this total split between the two groups? Oh, I can't sleep without finding out! This is more exciting than that *Melrose Place* cliffhanger!

First let's determine marginal cost:
MC $= 2Q = 2(411.111) =$ **822.222**

To figure out Q_e, let's use the rule $MR_e = MC$
$MR_e = MC$
$1000 – 8Q_e = 822.222$ so **$Q_e = 22.222$**

To figure out Q_y, let's use thy rule $MR_y = MC$
$MR_y = MC$
$1600 – 2Q_y = 822.222$ so **$Q_y = 388.888$**

But wait—what price should the firm charge to each group? Oh, this is better than a cartoon with Lippy the Lion and Hardy Har Har!

Use the elderly demand curve to calculate P_e

$\mathbf{P_e} = 1000 - 4Q_e$ **= $911.111**

Use the young demand curve to calculate Py

$\mathbf{P_y} = 1600 - Q_y$ **= $1211.111**

But wait—what's the firm's total revenue? Oh, this is better than watching Titanic with Barney the Lovable Pink Dinosaur!

total revenue = $P_eQ_e + P_yQ_y$ = $911.111(22.222) + $1211.111(388.888)

= 20246.70 + 470986.53 = $491233.23

But wait—what is the firm's total profit? Oh, this is better than being stranded in Cancun with Ru Paul!

total profit = total revenue – total cost

= $491233.23 – (100 + Q^2)$ = $491233.23 – (100 + (411.11^2))$

= $322,121