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Game Theory Basics

Contents:

Game Theory: An Introduction

Game theory had its genesis as a tool for gamblers to maximize their gain (or minimize their loss). In economics, one can model a market as a game, in which the players are the firms, and the object of the game is to maximize profits. One must impose a set of rules on the game; for example, one can prohibit collusion among players (prohibit a cartel). The course of the game and its ultimate outcome depend upon the strategies taken by the players and the rules imposed upon them; the course and outcome act as predictors of what might really happen in the market that one is modeling. (All of this can be done with mathematics.)

Nash Equilibrium:

One way to predict the outcome of a game is to find a situation in which each player is doing as well as it can, *given what the other players are doing* (and given the restrictions imposed on the actions of the players). This means that there is no incentive for any player to change what it is doing; we have an equilibrium, known as a *Nash equilibrium*.

Types of Games:

Repeating vs. non-repeating: If you play golf only once in your life, then it's a non-repeating game. If you play more than once, it's a repeating game. (In the notes file mana-imperfect, all of the duopoly examples—Cartel, Cournot, Stackelberg, Bertrand—were non-repeating games.)

Sequential vs. non-sequential: If players act in turn, then it's sequential. If players act simultaneously, it's non-sequential. Golf is a sequential game. Rock vs. scissors vs. paper is a non-sequential game. (In the notes file mana-imperfect, Stackelberg was sequential; Cartel, Cournot, and Bertrand were non-sequential.)

Cooperative vs. non-cooperative: If players can cooperate according to binding contracts, then it's cooperative. If not—non-cooperative. (In the notes file mana-imperfect, cartel was cooperative, all others non-cooperative.) NOTE! Nash equilibrium is restricted to a non-cooperative equilibrium.

Games can be combinations of the above. Gary Kasparov plays repeating, sequential, non-cooperative chess.

Object: maximize profits

In economics, one usually assumes that the goal of a player—a firm in a market—is to maximize profits. Let's look at examples below.

Nash equilibrium profit maximization:
Consider two firms in a duopoly--A and B--deciding whether or not to end a rebate promotion. (This is a non-repeating, non-sequential, non-cooperative game.) There are four possible future states of the world:

1) A ends rebate, B ends rebate
2) A ends rebate, B doesn't end it
3) A doesn't end rebate, B ends rebate
4) A doesn't end rebate. B doesn't end rebate

Let's use a **payoff matrix** to show how each firm's profits hinges on its decision combined with the decision of its rival firm. Inside each box in the payoff matrix are numbers (that I have made up for this example); each number represents a dollar amount of profits. The number before the comma is firm B's profits; the number after the comma is firm A's profits.

		firm A	
		ends rebate	doesn't end rebate
firm B	ends rebate	19,20	-5, 25
	doesn't end rebate	24, -6	1,2

Let's find a Nash equilibrium:

1) A ends rebate, B ends rebate. Nash equilibrium? NO!

Firm A is not doing as well as it can, *given that firm B has ended the rebate*. Firm A is making $20, but they could have made $25, if only they had chosen the "doesn't end rebate" option. The "doesn't end rebate" would have been a better option for firm A, *given that firm B has ended the rebate.*

Similarly, Firm B is not doing as well as it can, *given that firm A has ended the rebate*. Firm B is making $19, but they could have made $24, if only they had chosen the "doesn't end rebate" option. The "doesn't end rebate" would have been a better option for firm B, *given that firm A has ended the rebate.*

2) A ends rebate, B doesn't end it. Nash equilibrium? NO!

Firm A is not doing as well as it can, *given that firm B hasn't ended the rebate*. Firm A is losing $6, but they could have made $2, if only they had chosen the "doesn't end rebate" option. The "doesn't end rebate" would have been a better option for firm A, *given that firm B hasn't ended the rebate.*

3) A doesn't end rebate, B ends rebate. . Nash equilibrium? NO!

Firm B is not doing as well as it can, *given that firm A hasn't ended the rebate*. Firm A is losing $5, but they could have made $1, if only they had chosen the "doesn't end rebate" option. The "doesn't end rebate" would have been a better option for firm B, *given that firm A hasn't ended the rebate.*

4) A doesn't end rebate. B doesn't end rebate Nash equilibrium? Yes!

Firm A is doing as well as it can, *given that firm B hasn't ended the rebate*. Firm A is making $2, but they would have lost $6, if they had chosen the "end rebate" option. The "end rebate" would have been a worse option for firm A, *given that firm B hasn't ended the rebate.*

Similarly, Firm B is doing as well as it can, *given that firm A hasn't ended the rebate*. Firm B is making $1, but they would have lost $5, if they had chosen the "end rebate" option. The "end rebate" would have been a worse option for firm B, *given that firm A hasn't ended the rebate.*

Conclusion: It may be wise for neither firm to end the rebate, because if one firm ends the rebate while its competitor does not then that firm will be worse off.

Cooperative equilibrium profit maximization:
Using the same payoff matrix and the same example from above, we can see that option (1) is a cooperative equilibrium. If both firms end the rebate, both of them do better than if they competed with each other (and the Nash equilibrium resulted).

Conclusion: Collusion often results in higher profits than competition. If the two firms could meet and agree that they would both end the rebate, then both firms will be better off. Such agreements, however, are generally illegal in the U.S. (in order to protect consumers).

Implication from above: Have you ever wondered why all of these ridiculous rebates exist for autos? Why don't they just cut the price of the autos, and forget about these danged rebates? Well, one reason may be that each individual automaker fears that if it ends its rebates then its competitors will not end theirs, so the individual firm will be made worse off. If somehow the automakers were allowed to collude, then they might all agree to simultaneously end their rebates.

A cooperative equilibrium is possible if (1) the rules of the game allow it and (2) there exists at least one outcome that is superior for all players to a Nash equilibrium.

Risk Aversion and the **Maximin** Strategy:
No future outcome is certain; there is risk to any strategy. Some managers may be *risk averse*; they may want to avoid at all costs a strategy that could result in a terrible loss (or a miniscule gain).

(Are you risk averse? Which would you prefer: a $1000 prize, or a 1-in 999 chance to win $1,000,000? If you chose the $1,000 then you are risk averse.)

Maximin means "maximize the minimum gain." In this case, the player picks the choice whose *worst possible outcome is the least bad*.

Example:

Ford must choose which new car to introduce:

(a) Ford introduces the new "Wacky Car!" it may make as much as $1 billion or as little as $10 million, depending on how the public takes to this new wacky car.

(b) Ford introduces the restyled Taurus, a proven but dull car, it may make as much as $100 million or as little as $50 million.

The profit-maximizing strategy is probably (a), since the potential for profits is so great. But this choice is risky, since profits could be as little as $10 million.

The maximim strategy is definitely (b), since the worst possible outcome of (b)—a profit of $50 million, is better than the worst possible outcome of (a)—a profit of $10 million.

> The maximin choice is usually not the profit-maximizing choice, and is a sign that management ought to be replaced by folks willing to maximize the return to shareholders.

Here is a payoff matrix example of the maximin equilibrium. Here, both firms make the choice where the worst possible outcome is the least bad.
Two firms must decide: Accept competitors' coupons, or not?

		firm 1	
		accept coupons	not
firm 2	accept coupons	1,2	5,1
	not	-1,10	7,12

The least risky strategy for firm 1 is to accept coupons, because the worst possible outcome in that case—a profit of $2—is not as bad as the worst possible outcome if they did not accept coupons—a profit of $1.

The least risky strategy for firm 2 is also to accept coupons, because the worst possible outcome in that case—a profit of $1—is not as bad as the worst possible outcome if they did not accept coupons—a loss of $1.

Hence the maximin equilibrium is for both to accept coupons. Firm 1 makes $2 and firm 2 makes $1.

Notice how this outcome is inferior in terms of pure profits to the bottom right choice in the matrix. Risk aversion has a price—lower long term profits.

<u>Special Game: Dominant Strategy Equilibrium:</u>

In rare cases, a firm's best choice may be clear, if it always does better with that choice regardless of the choices made by its rivals. If this is true then the firm has a *dominant strategy*.

Example: Suppose Continental Airlines has a ton of unsold seats during a slow period of travel. It must decide whether or not to cut fares. There are 3 possible future outcomes:
1) It doesn't cut fares

2) It cuts fares and its competitors match the price cuts
3) It cuts fares and its competitors don't match the price cuts.

Suppose Dr. Big Shot, economist, has profit projections for Continental for all 3 future outcomes:
1) Profit = $1,000,000
2) Profit = $2,000,000
3) Profit = $10,000,000

You see how the profit is always higher if the airline cuts its fares, regardless of how its competitors respond? The dominant strategy is to cut fares.

Let's do a payoff matrix example of a dominant strategy equilibrium. There are two stores—Wal-Mart and K-Mart. They are trying to decide whether to give bad customer service, or to spend more money and have good customer service.

		Wal-Mart	
		bad service	good service
K-Mart	bad service	20,30	10,80
	good service	19,5	2,50

You see how bad service is the dominant strategy for K-mart? They will make more profits under that strategy, no matter what Wal-Mart does.

You see how good service is the dominant strategy for Wal-mart? They will make more profits under that strategy, no matter what K-Mart does.

Hence the dominant strategy equilibrium is bad service by K-mart and good service by Wal-Mart.

A Final Crack at These Concepts

Here's one more example for you to practice some of the concepts in these notes.

Two firms are trying to decide to set a high price or a low price for their product.

		firm 1	
		low price	high price
firm 2	low price	-10,-9	20,-18
	high price	-30,2	30,5

There is no dominant strategy for either firm.

The maximin equilibrium: firm 1 low price, and firm 2 low price

There are two Nash equilibrium:

(1) firm 1 low price, and firm 2 low price

(2) firm 1 high price, and firm 2 high price

There is one cooperative equilibrium: firm 1 high price, and firm 2 high price

Please be sure that you can figure out all of the above answers. You may have to reread and study the notes for a while before you can do this. This is normal, believe me. This is a relatively short set of notes, but you may be staring at it and scratching your head for a while.