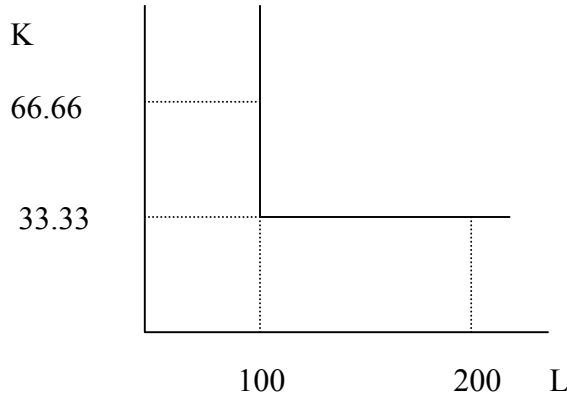


**Answers to Second Midterm (Modified 4/6/2003)**

1. A firm has production function  $Q = \min[3K, L]$ . K costs \$20 per unit; L costs \$5 per unit.

a) Graph this firm's isoquant for 100 units of output. Include at least 3 points on your isoquant, and label the numerical coordinates of the 3 points.



b) Calculate the cost-minimizing use of capital and labor.

33.333 units of K and 100 units of L

c) Is this firm's production consistent with the Law of Diminishing returns? Explain carefully, and support your answer with at least 1 calculation.

No. the marginal product of one factor is constant at zero; it does not, as the law states, diminish as more of the factor is employed.

Example: at  $K=33.33$  and  $L=100$ ,  $Q=100$ . Now add 1 unit of  $L \rightarrow Q$  remains at 100. Now add another unit of  $L \rightarrow Q$  remains at 100. The marginal product of labor is constant at zero.

d) Is this firm's production consistent with increasing returns to scale? Explain carefully, and support your answer with at least 1 calculation.

NO. Doubling factor use does not more than double output. Example: Double K and L from 33.33 and 100 respectively, to 66.66 and 200, respectively. Output exactly doubles to 200.

2. A perfectly competitive market has demand curve  $Q = 1000 - 5P$  and supply curve  $Q = 5P$ . An individual seller in this market has cost curve  $C = 200 + 5Q^2$

a) Calculate the individual seller's profit-maximizing price, quantity and profit (or loss). (Please be clear whether the firm has a profit or a loss.)

Market price is the firm's marginal revenue; find P by setting demand equal to supply.  
 $1000 - 5P = 5P \rightarrow P = 100 \rightarrow MR = 100$

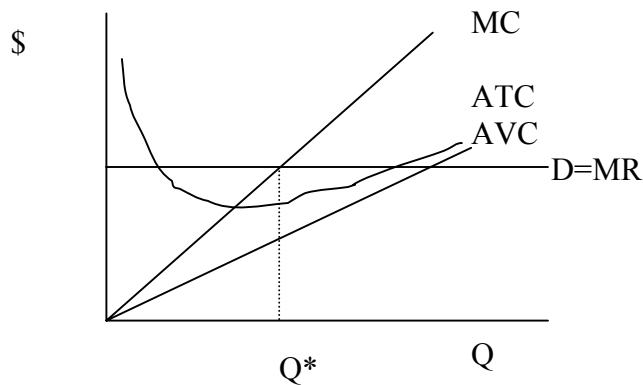
MC is derivative of cost equation with respect to Q       $MC = 10Q$

Set firm's  $MR = MC$        $100 = 10Q \rightarrow Q = 10$

Profit = revenue - cost =  $PQ - C = 100(10) - [200 + 5(10^2)] = 1000 - 700 = 300$

Price 100      Quantity 10      Profit (or loss) profit of 300

b) Carefully illustrate this firm's situation on a graph that includes its demand, marginal revenue, marginal cost, average total cost, and average variable cost. (Number are not required on this graph, nor is a precise scale. Please take care with the SHAPES of the curves. Some of the curves may have different shapes than those encountered in class.)



3. A monopoly has

demand curve

$$Q = 100 - 5P \text{ and}$$

AVERAGE cost curve

$$\text{average cost} = 5Q$$

a) Calculate the firm's profit-maximizing price and quantity.

Rearrange demand to get P on the left:  $P = 20 - .2Q$

Marginal revenue is twice as steep as demand  $MR = 20 - .4Q$

Marginal cost is derivative of total cost. total cost = average cost  $\times Q = 5Q(Q) = 5Q^2$

So marginal cost =  $10Q$

Set  $MR = MC$   $20 - .4Q = 10Q \rightarrow 10.4Q = 20 \rightarrow Q = 1.923$  (rounded)

Get price by using demand curve:  $P = 20 - .2(1.923) = 19.615$  (rounded)

Price 19.615

Quantity 1.923

b) Calculate the price ceiling required to ensure social efficiency.

$P = MC \rightarrow 20 - .2Q = 10Q \rightarrow 10.2Q = 20 \rightarrow Q = 1.9608$  (rounded)

Plug this Q into demand (or MC):  $P = 20 - .2(1.9608) \rightarrow P = 19.607$

Ceiling 19.607

c) Does this firm require a subsidy in order to be socially efficient? Support your answer with at least 1 calculation.

No. The ceiling of 19.607 exceeds average cost, since average cost =  $5(1.9608) = 9.804$

4. A firm has demand for its product equal to

$$Q = 500 - 2P$$

It can produce the product in either of two plants, with total costs

$$C_1 = 100 + Q_1^2 \qquad C_2 = 10 + 2Q_2^2$$

Calculate the profit maximizing level of production in plants 1 and 2, the profit-maximizing price, and total profits (or loss).

Rearrange demand to get P on the left:  $P = 250 - .5Q$   
 MR is twice as steep  $\mathbf{MR = 250 - Q}$

MC for plant 1:  $MC_1 = 2Q_1$  or  $Q_1 = .5MC_1$   
 MC for plant 2:  $MC_2 = 4Q_2$  or  $Q_2 = .25MC_2$

Horizontally add MCs  $Q = .75MC$   
 Rearrange to get MC on left  $\mathbf{MC = 1.333Q}$  (rounded)

Set MR = horizontal sum of MC:  
 $250 - Q = 1.333Q \rightarrow 2.333Q = 250 \rightarrow Q = 107.1429$  (rounded)

At this Q,  $MR = 250 - 107.1429 = 142.8571$  (rounded)

Substitute this MR into MC equations to get  $Q_1$  and  $Q_2$

$$Q_1 = .5(142.8571) = 71.4286 \text{ (rounded)}$$

$$Q_2 = .25(142.8571) = 35.7143 \text{ (rounded)}$$

Substitute  $Q = 107.1429$  into demand equation to get price :

$$P = 250 - .5(107.1429) = 196.4286 \text{ (rounded)}$$

$$\text{Profit} = \text{revenue} - \text{cost} = PQ - C_1 - C_2$$

$$= 196.4286(107.1429) - [100 + (71.4286)^2] - [10 + 2(35.7143^2)]$$

$$= 21100.7281 - 5202.0449 - 2561.0224 = 13282.8265 \text{ (rounded)}$$

$$Q_1 \underline{\hspace{1cm}} 71.4286$$

$$Q_2 \underline{\hspace{1cm}} 35.7143$$

$$\text{Price} \underline{\hspace{1cm}} 196.4286 \quad \text{Profits } 13282.8265$$

5. Explain whether each statement below is true or false. Do not repeat the statement or a slightly altered version of it in your explanation. Use concepts developed in this class in your explanation. Merely writing “true” or “false” results in zero credit.

a) An owner of a perfectly competitive firm will live in poverty in the long run, since her/his firm will have zero profits.

False. Since implicit costs are included in the cost calculation, the owner will earn income equivalent to what he would earn using his skills working for someone else.

b) A firm maximizes total profits at the level of output at which price exceeds average total cost by the largest amount.

False. This statement suggests maximization of a profit margin. But to maximize total profit the firm should produce where  $MR = MC$ .

c) A firm should shut down if its revenues are less than its fixed costs.

False. If a firm cannot earn enough revenue to pay even for its variable costs, then it should shut down, since it will suffer a smaller loss in this case when shut down.

d) The “haircut” market is an example of a market in which production exhibits economies of scale.

False. Average costs are roughly constant in this industry, as evidenced by the fact that smaller firms (the local barber) have no trouble competing with larger ones (Supercuts).

e) Older, established firms are more likely to benefit in the future from the “learning curve” effect than are newer firms (of the same size as the older firms).

False. There are diminishing returns to learning. A new firm has greater opportunity to learn how to reduce its costs, since it hasn’t learned anything yet. (Analogy: it’s easier for a new golfer to lower his/her score substantially than it is an experienced golfer.)