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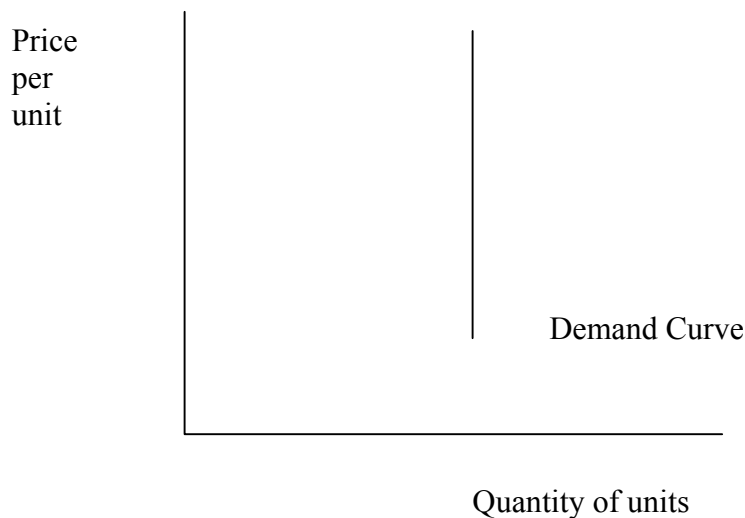
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DEMAND AND STEEPNESS

Why is one demand curve steeper than another? The steepness of a demand curve is a measure of the responsiveness of buyers to a change in price of the product. The steeper a demand curve is, the less responsive buyers are to a change in price.

Consider, for example, the hypothetical extreme of a completely vertical demand curve:



These buyers do not change the quantity that they demand when the price changes. An approximate real world example is insulin; a diabetic needs it to survive, and will continue to buy the same quantity even if the price skyrockets.

Here are some things that affect steepness of a demand curve:

1. Number of substitutes for the product

If a product has few substitutes, then buyers have few options when the price of the product rises; hence they are likely to keep buying the product, though perhaps in slightly lower quantities. Such a product will have a pretty steep demand curve.

On the other hand, if a product has many substitutes, then it is easy for buyers to stop buying the product when the product's price rises, switching to substitute products; hence a product with many substitutes will have a relatively flat demand curve.

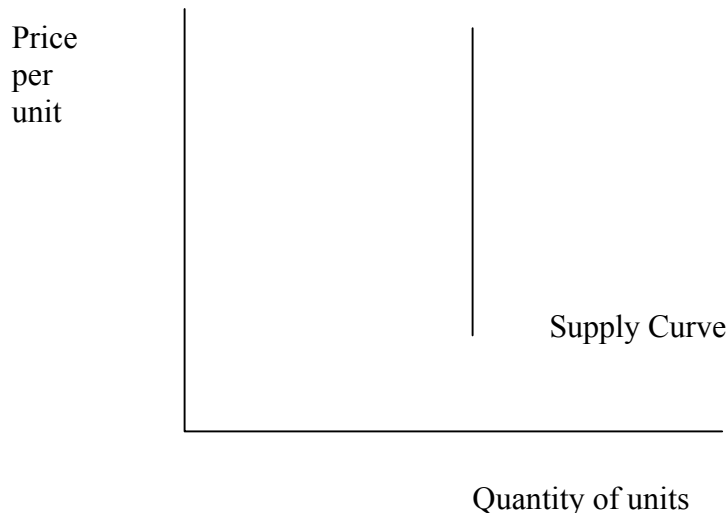
2. Time

Suppose gasoline prices skyrocket. Over the short term, you may be forced to buy almost as much gas as at lower prices, since you may need to drive to get to work, etc. Over a longer term, however, you could buy a more fuel efficient vehicle, and reduce your gas consumption by a larger amount. Conclusion: for most (not all) products, more time implies a flatter demand curve.¹

SUPPLY AND STEEPNESS

Why is one supply curve steeper than another? The steepness of a supply curve is a measure of the responsiveness of sellers to a change in price of the product. The steeper a supply curve is, the less responsive sellers are to a change in price.

Consider, for example, the hypothetical extreme of a completely vertical supply curve:



These sellers do not change the quantity that they sell when the price changes. An approximate real world example is harvested bananas; the bananas are already harvested, and sellers will want to sell them all regardless of the price they can get for them (because spoiled unsold bananas do not add to revenue).

Time is the most important thing that affects the steepness of a supply curve. Let us define and consider three time periods:

¹ Goods which are *durable*—benefiting the consumer for a few years (such as autos)—actually have steeper demand curves given more time. Suppose lawn mower prices skyrocket, for example. Over the short term buyers may delay acquisition of new mowers to replace old ones, hoping that prices will fall back down; this means a very flat demand curve. Over the long term, however, lawn mowers wear out, and buyers must replace them; this means a steeper demand curve.

1. The Market Period

In the market period, sellers do not have enough time to alter that quantity of product available for sale. The banana example above is a market period example. The supply curve is *vertical* in the market period.

2. The Short Run

In the short run, sellers are able to alter the quantity supplied, but they have *at least one fixed factor of production*. Suppose a banana farm believes that bananas will fetch a high price next year. This is enough time for the banana farmer to spend a little more money to take extra special care of the banana trees, to ensure a slightly higher banana yield, taking advantage of the higher banana prices. But the banana farmer doesn't have enough time to plant new banana trees that will yield fruit next year. This is a short run example; the trees are the farmer's fixed resource.

The supply curve is steep, but not vertical, in the short run.

3. The Long Run

In the long run, sellers are able to alter the quantity supplied, and they have *no fixed factors of production*. Suppose a banana farm believes that bananas will fetch a high price in 30 years. This is enough time for the banana farmer to spend a more money planting extra banana trees, to ensure a massively higher banana yield in 30 years, taking great advantage of the higher banana prices. This is a long run example; the trees are the now a variable resource.

The supply curve is very flat in the long run; it may even be downward-sloping. (More on this in later notes.)

FROM STEEPNESS TO ELASTICITY

Sure, steepness is an indicator of the responsiveness of quantity to changes in price, but it's quite vague, ain't it? *Price elasticity* is a much more precise measure of this price/quantity relationship. We shall discuss price elasticity, and some other elasticities, in the next section of notes.

ELASTICITY

General: *Elasticity* is a ratio of the percentage change in one variable to the percentage change in another variable. We shall discuss four elasticities below:

(1) Price elasticity of demand =

percentage change in quantity demanded / percentage change in price

(2) Price elasticity of supply =

percentage change in quantity supplied / percentage change in price

(3) Income elasticity of demand =

percentage change in quantity demanded / percentage change in income

(4) Cross elasticity of demand =

percentage change in quantity demanded of good A / percentage change in price of good B

More about price elasticity of demand:

Example 1: Suppose a 10% price reduction in Coke causes a 5% increase in unit sales. Then the price elasticity of demand of coke is

$$5\% / -10\% = -0.5$$

Example 2: Suppose a 2% increase in pretzel prices causes a 10% reduction in unit sales. Then the price elasticity of demand of pretzels is

$$-10\% / 2\% = -5$$

From the above examples we can note that:

Price elasticity of demand is almost always negative, but sometimes by convention we take the absolute value of the elasticity when expressing it.

The closer the price elasticity of demand is to zero, the less responsive is demand to changes in price.

Calculating price elasticity of demand using the *Arc* formula:

Example 3: Suppose the following points are on a hypothetical demand curve:

Point A: At a price of \$10 quantity demanded is 100

Point B: At a price of \$20 quantity demanded is 30

We can calculate the price elasticity of demand (E_d) for the segment of the demand curve between points A and B using the Arc formula:²

$$E_d = \frac{Q_A - Q_B}{(Q_A + Q_B)/2} \div \frac{P_A - P_B}{(P_A + P_B)/2}$$

² Another formula which can be used to calculate elasticity—the *point* formula—is discussed a bit later in these notes

$$= \frac{100 - 30}{(100 + 30)/2} \div \frac{10 - 20}{(10 + 20)/2}$$

$$= 1.076923076923 / -.6666666666 = -1.615384615386$$

by convention, we eliminate the minus sign: $E_d = 1.615384615386$

Price Elasticity of Demand: The Point Formula

The arc formula, I hope, makes intuitive sense to you. It is of great interest to a firm, if it changes its price by a certain realistic percentage, to know by what percentage quantity demanded will change.

But suppose that you want to know the percentage that quantity demanded will change for a very small change in price—indeed, for an infinitesimally small change in price. Can we calculate such an elasticity? Yes, with the **point** formula. Let's derive (sort of) this formula:

Remember, generally, that price elasticity of demand represents percentage change in quantity demanded divided by percentage change in price:

$$E_d = \% \text{ change in } Q_d / \% \text{ change in } P$$

How can we express % change in Q_d ? Well, how about this:

$$\% \text{ change in } Q_d = \text{change in } Q_d / Q_d$$

How can we express % change in P ? Well, how about this:

$$\% \text{ change in } P = \text{change in } P / P$$

Let's substitute these expressions into our elasticity equation:

$$E_d = \frac{\text{change in } Q_d}{Q_d} \div \frac{\text{change in } P}{P}$$

We can rearrange this equation to look like this:

$$E_d = \frac{\text{change in } Q_d}{\text{change in } P} \times \frac{P}{Q_d}$$

Now, suppose that we want to calculate E_d for an infinitesimally small change in P . Well, that's our friendly derivative from calculus! dQ_d / dP represents the change in Q_d when P changes an extremely small amount. Let's substitute this derivative into the elasticity equation:

$$E_d = \frac{\partial Q_d}{\partial P} \times \frac{P}{Q_d}$$

Well, shucks ma'am—that there's the point elasticity of demand formula!

Using the point formula: example 4

Suppose we have a demand equation: $Q_d = 100 - 5P$

Let's calculate the price elasticity of demand at a price of \$15

Note: $P = \$15$

Now, plug in $P = \$15$ into the demand equation to get Q_d :

$$Q_d = 100 - 5(15) = 25$$

Now, calculate dQ_d / dP for our demand equation:

$$dQ_d / dP = -5$$

Now, substitute all of our calculations into the point elasticity formula:

$$E_d = -5 \times \frac{15}{25} = -3 \quad \text{or } 3 \text{ by convention}$$

Price Elasticity of demand and total revenue

Define *total revenue* (TR) as the total dollar value of a firm's sales. For a firm that only produces one product (and sells it at one price per unit), this is equal to price per unit times quantity bought:

$$TR = PQ$$

A firm might want to know how changing its product's price per unit affects its total revenue. Well, guess what? The value of the price elasticity of demand tells the firm something about how a price change will affect total revenue!!

If E_d is greater than 1, then a higher price results in lower total revenue, and a lower price results in higher total revenue. Don't believe me? Check out example 3: raising the price from \$10 to \$20 causes total revenue to fall from \$1000 to 600; cutting the price from \$20 to \$10 causes total revenue to rise from \$600 to \$1000.

If E_d is between 0 and 1, then a higher price results in higher total revenue, and a lower price results in lower total revenue. In example 1, Coke revenue will rise if Coke's price is increased; Coke revenue will fall if Coke's price is reduced.

We're done with price elasticity of demand; now, on to price elasticity of supply!

MORE ABOUT PRICE ELASTICITY OF SUPPLY

Example 4: Suppose a 10% price reduction in wheat causes a 5% reduction in wheat supply. Then the price elasticity of supply of wheat is

$$-5\% / -10\% = .5$$

Example 5: Suppose a 2% increase in milk prices causes a 10% increase in milk supply. Then the price elasticity of supply of milk is

$$10\% / 2\% = 5$$

From the above examples we can note that:

Price elasticity of supply is usually positive

The closer the price elasticity of supply is to zero, the less responsive is supply to changes in price.

Calculating price elasticity of supply using the Arc formula:

Example 3: Suppose the following points are on a hypothetical supply curve:

Point A: At a price of \$10 quantity supplied is 30

Point B: At a price of \$20 quantity supplied is 100

We can calculate the price elasticity of demand (E_s) for the segment of the supply curve between points A and B using the Arc formula:³

$$E_s = \frac{Q_A - Q_B}{(Q_A + Q_B)/2} \div \frac{P_A - P_B}{(P_A + P_B)/2}$$

³ Another formula which can be used to calculate price elasticity of supply—the *point* formula—is virtually the same as the formula for point elasticity of demand—just use Q_s instead of Q_d in the formula.

$$= \frac{30 - 100}{(30 + 100)/2} \div \frac{10 - 20}{(10 + 20)/2}$$

$$= -1.076923076923 / -.666666666666 = 1.615384615386$$

That's it for price elasticity of supply. Now, on to income elasticity of demand!

MORE ABOUT **INCOME** ELASTICITY OF DEMAND (E_I)

Example 6: Suppose a 10% income reduction causes a 5% reduction in steak demand. Then the income elasticity of demand of steak is

$$-5\% / -10\% = .5$$

Example 7: Suppose a 2% increase in income causes a 10% reduction in ramen noodles demand. Then the income elasticity of demand of ramen noodles is

$$-10\% / 2\% = -5$$

From the above examples we can note that:

Income elasticity of demand (E_I) may be a positive or negative number

Define a *normal good* as one whose demand is positively associated with income. E_I is positive for normal goods. (Steak is normal in example 6)

Define an *inferior good* as one whose demand is negatively associated with income. E_I is negative for inferior goods. (Ramen noodles are inferior in example 7)

Income Elasticity of Demand: The **Point** Formula

Suppose that you want to know the percentage that quantity demanded will change for a very small change in income—indeed, for an infinitesimally small change in income. Can we calculate such an elasticity? Yes, with the **point** formula. Let's derive (sort of) this formula:

Remember, generally, that income elasticity of demand represents percentage change in quantity demanded divided by percentage change in income (I):

$$E_I = \% \text{ change in } Q_d / \% \text{ change in } I$$

How can we express % change in Q_d ? Well, how about this:

$$\% \text{ change in } Q_d = \text{change in } Q_d / Q_d$$

How can we express % change in I ? Well, how about this:

$$\% \text{ change in } I = \text{change in } I / I$$

Let's substitute these expressions into our elasticity equation:

$$E_d = \frac{\text{change in } Q_d}{Q_d} \div \frac{\text{change in } I}{I}$$

We can rearrange this equation to look like this:

$$E_d = \frac{\text{change in } Q_d}{\text{change in } I} \times \frac{I}{Q_d}$$

Now, suppose that we want to calculate E_I for an infinitesimally small change in I . Well, that's our friendly derivative from calculus! dQ_d / dI represents the change in Q_d when I changes an extremely small amount. Let's substitute this derivative into the elasticity equation:

$$E_d = \frac{\partial Q_d}{\partial I} \times \frac{I}{Q_d}$$

Well, shucks ma'am—that there's the point income elasticity of demand formula!

Using the point formula: example 5

Suppose we have a demand equation: $Q_d = 75 - 5P + .02I$

Let's calculate the price elasticity of demand at an income of \$200 and price of \$15

Note: $I = \$200$ and $P = 15$

Now, plug in $P = \$15$ and $I = \$200$ into the demand equation to get Q_d :

$$Q_d = 75 - 5(15) + .02(200) = 4$$

Now, calculate dQ_d / dI for our demand equation:

$$dQ_d / dI = .02$$

Now, substitute all of our calculations into the point elasticity formula:

$$E_I = .02 \times \frac{200}{4} = 1$$

So much for income elasticity of demand. Now, on to cross elasticity of demand!

MORE ABOUT **CROSS** ELASTICITY OF DEMAND (E_C)

Example 8: Suppose a 10% increase in Coke prices causes a 5% increase in Pepsi demand. Then the cross elasticity of demand of Coke and Pepsi is

$$5\% / 10\% = .5$$

Example 9: Suppose a 2% increase in hot dog prices causes a 10% reduction in hot dog bun demand. Then the cross elasticity of demand of hot dogs and hot dog buns is

$$-10\% / 2\% = -5$$

From the above examples we can note that:

Cross elasticity of demand (E_C) may be a positive or negative number

Define *substitute* goods as those that serve roughly the same purpose to buyers. E_C is positive for substitute goods. (Coke and Pepsi are substitutes in example 8)

Define *complementary* goods as those that are often consumed together. E_C is negative for complementary goods. (Hot dogs and hot dog buns are complements in example 9)

Cross Elasticity of Demand: The **Point** Formula

Suppose that you want to know the percentage that quantity demanded of one good will change for a very small change in price of another good—indeed, for an infinitesimally small change in price. Can we calculate such an elasticity? Yes, with the **point** formula. Let's derive (sort of) this formula:

Remember, generally, that cross elasticity of demand (E_C) represents percentage change in quantity demanded of good X divided by percentage change in price of good Y:

$$E_C = \% \text{ change in } Q_x / \% \text{ change in } P_y$$

How can we express % change in Q_x ? Well, how about this:

$$\% \text{ change in } Q_x = \text{change in } Q_x / Q_x$$

How can we express % change in P_y ? Well, how about this:

$$\% \text{ change in } P_y = \frac{\text{change in } P_y}{P_y}$$

Let's substitute these expressions into our elasticity equation:

$$E_{xy} = \frac{\text{change in } Q_x}{Q_x} \div \frac{\text{change in } P_y}{P_y}$$

We can rearrange this equation to look like this:

$$E_{xy} = \frac{\text{change in } Q_x}{\text{change in } P_y} \times \frac{P_y}{Q_x}$$

Now, suppose that we want to calculate E_c for an infinitesimally small change in P_y . Well, that's our friendly derivative from calculus! dQ_x / dP_y represents the change in Q_x when P_y changes an extremely small amount. Let's substitute this derivative into the elasticity equation:

$$E_c = \frac{\partial Q_x}{\partial P_y} \times \frac{P_y}{Q_x}$$

Well, shucks ma'am—that there's the point cross elasticity of demand formula!

Example: Suppose we have a demand equation for good X:

$$Q_x = 1000 - 5P_x + 2P_y$$

Suppose also that P_x is currently \$10 and P_y is currently \$15

Now, plug in $P_x = \$10$ and $I = \$200$ into the demand equation to get Q_d :

$$Q_x = 1000 - 5(10) + 2(15) = 980$$

Now, calculate dQ_x / dP_y for our demand equation:

$$dQ_d / dP_y = 2$$

Now, substitute all of our calculations into the point elasticity formula:

$$E_C = 2 \times \frac{15}{980} = .0306$$

Conclusion:

Managers find elasticities useful because elasticities provide information on how a change in one thing affects another thing. This information can be very valuable for planning purposes. For example, Intel corporation recently cut prices on its microprocessors; so how many microprocessors should Intel produce this month? Well, if Intel has a good estimate of the price elasticity of demand for its product, then it will have a good idea how much to change production, because it has a sense of how the price change will affect the quantity of microchips that its customers will buy.

Those decision makers (such as MBAs) who can calculate, understand, and/or interpret the meaning of a company's elasticities are quite valuable assets to the company and will be well compensated. (Generally, an economist or other specialist will calculate the elasticities and give these values to managers; it is up to the managers to use the information wisely.)