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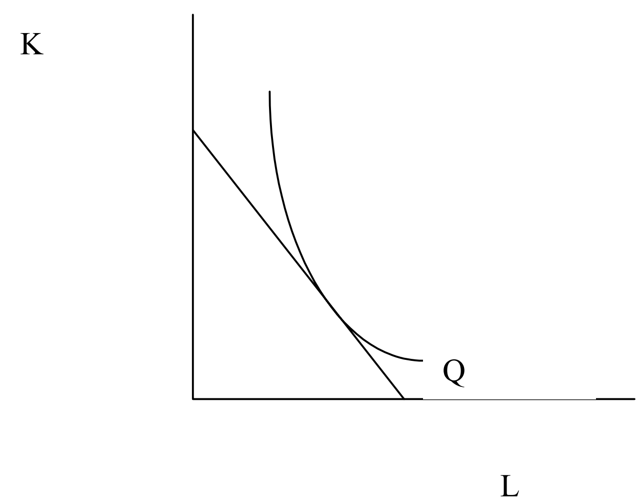
Costs of Production

Contents:

Long Run Costs

Effect of a change in a factor price: Recall from previous notes that, for a given price of labor (w) and price of capital (r), most firms minimize costs of producing a level of output Q by setting the ratio of factor prices (w/r) equal to the ratio of marginal products of the factors (MP_L/MP_K). Graphically this is equivalent to a tangency of an isoquant to a budget constraint, as illustrated below:



One might ask: how should the firm's employment of K and L change if the price of one of the factors changes?

<u>Example</u> 1: Suppose w falls, making L cheaper to hire. What is the new cost-minimizing use of K and L? (Will the firm hire *more* or *less* L? *More* or *less* K?)

Use of L: the law of demand tells us that the firm will almost certainly employ more L, since it has become cheaper relative to K.

Use of K: you may be tempted to believe that use of K will fall, since it has become more expensive relative to L, but *use of K may not fall*!!!! Let's examine this issue more closely below.

Economists point to two effects that work against each other in determining how much K is employed after a change in w:

1. The *factor substitution effect*: This is the more intuitive effect. Since L has become cheaper relative to K, producers tend to substitute L for K, reducing their use of K. the factor substitution effect measures this substitution—more L, *less K*.

2. The *output effect*: Cheaper L reduces costs of producing the firm's product. This allows the firm to lower the product's price. (Indeed, if they have intense competition then they must lower the price.) As a result of the lower price, unit sales of the product rise. The firm must produce more of the product, increasing their need for all factors of production, including K! The output effect measures this increase in K use—more output, *more K*.

The overall effect of a wage reduction on the use of K is theoretically ambiguous, since it is a combination of the factor substitution effect and the output effect, and those two effects work against each other.

HOW QUICKLY DO COSTS RISE AS Q RISES?

Suppose a firm follows the cost minimizing rule for whatever level of output that it produces; this means that it produces any level of output using the cheapest possible combination of factors. Still, the firm's total costs will rise as its level of production (Q) rises—it is simply a law of physics that more production requires more factors, no matter how frugal the firm is.

So, one important piece of information for the firm is: how quickly will their costs rise as Q rises? We shall discuss this issue in the pages that follow, for 2 time periods—the long run and the short run. But first, a few ways to measure costs:

Total them up: Add up the costs.

Average the costs: Spread the costs evenly over each unit produced. Total/Q = Average

Marginal costs: A measure of how quickly costs increase. This is the increase in total costs when 1 additional unit of Q is produced. More formally, it is $\frac{\Delta total}{\Delta Q}$. Even more formally, let's use calculus. Let "TC" represent total cost. Then marginal cost = $\frac{\partial TC}{\partial Q}$

<u>A 2-PAGE DIGRESSION: Relationship between marginal and average costs:</u>

--When average costs are *falling*, marginal costs are lower than average costs. (Indeed, it is the low marginal cost that is pulling the average down.)

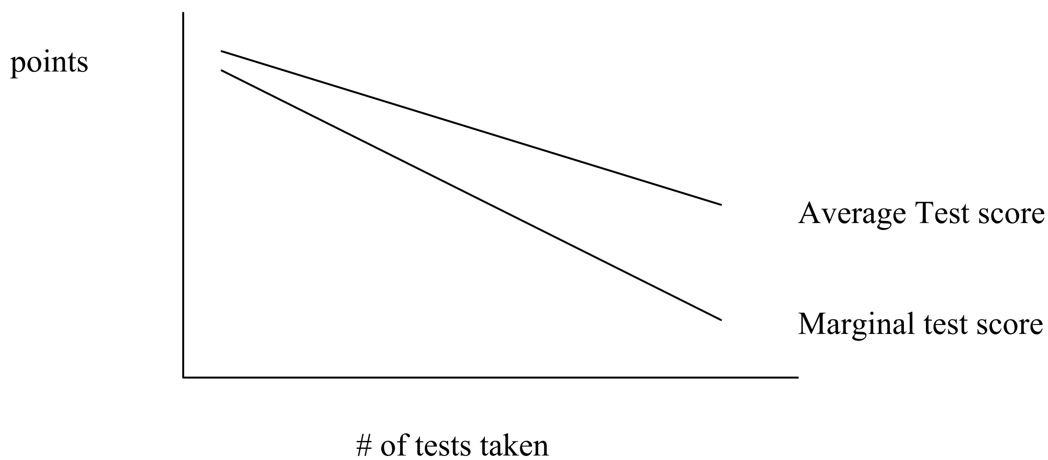
--When average costs are *rising*, marginal costs are higher than average costs. (Indeed, it is the high marginal cost that is pulling the average up.)

Let's illustrate the relationship between marginal and average using an example not of costs, but of test scores of four different students. Zippy keeps getting lower and lower scores on his tests. Buffy keeps getting higher and higher scores. Dullorama keeps getting the same test score over and over. Clutch began the semester getting lower and lower test scores, but finished up the semester getting higher and higher test scores.

Here's a table showing Zippy's lower and lower test scores (in the middle column), along with his test average (in the right hand column):

Marginal test	**Marginal test score**	**Zippy's test average**
1st	100	100
2nd	90	95
3rd	80	90
4th	70	85
5th	60	80
6th	50	75
7th	40	70

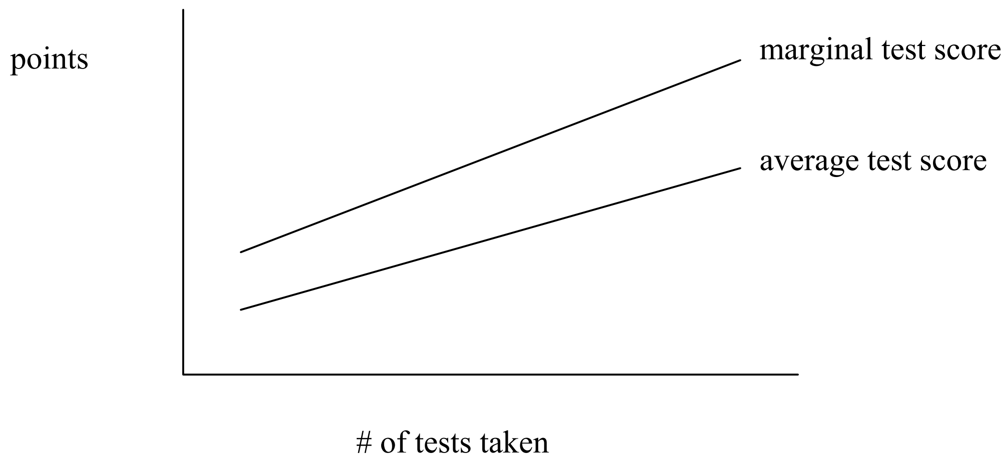
You see? When Zippy's average falls, it's because his marginal test score is below his average. If we were to graph Zippy's pathetic test performance, it would look something like this. (Graph not to scale)



Now here's a table showing Buffy's higher and higher scores, and her rising average:

Marginal test	**Marginal test score**	**Buffy's test average**
1st	30	30
2nd	40	35
3rd	50	40
4th	60	45
5th	70	50
6th	80	55
7th	90	60

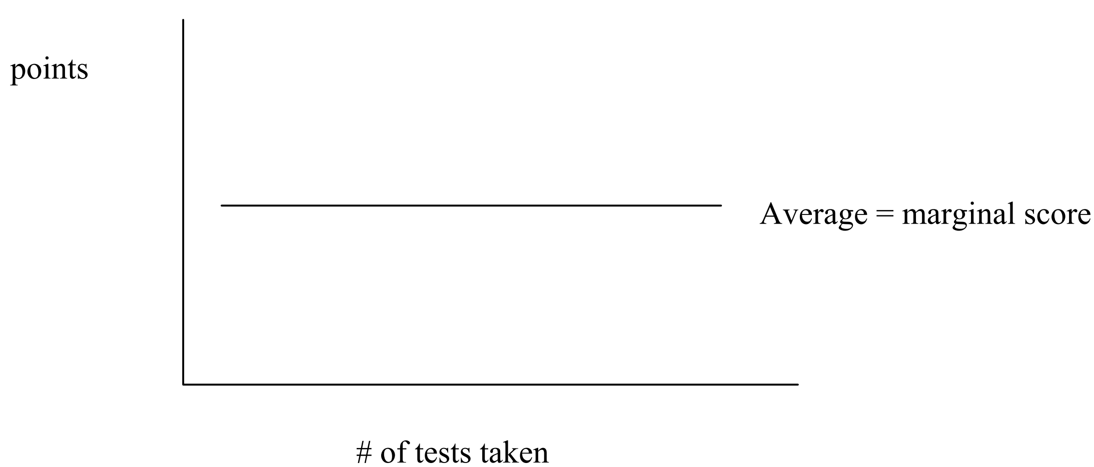
You see? When Buffy's average rises, it's because her marginal test score is above her average. If we were to graph Buffy's rising test performance, it would look something like this.
(Graph not to scale)



Now here's a table showing Dullorama, who keeps getting 80s on tests:

Marginal test	Marginal test score	Dullorama's test average
1st	80	80
2nd	80	80
3rd	80	80
4th	80	80
5th	80	80
6th	80	80
7th	80	80

You see? Since Dullorama's marginal test score equals his average, his test average remains constant. Dullorama is graphed below:
(Graph not to scale)

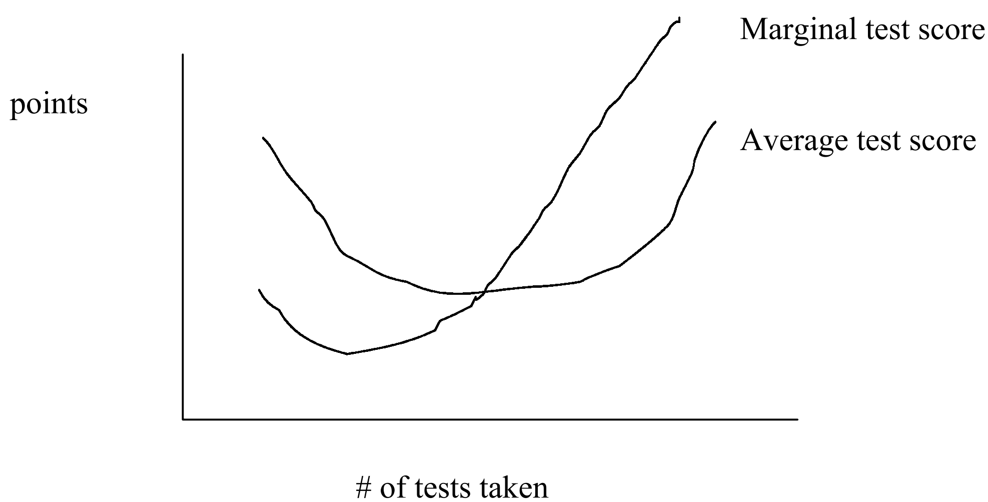


Finally, here's a table showing Clutch, whose scores dipped in the middle of the semester but who recovered nicely:

Marginal test	**Marginal test score**	**Clutch's test average**
1st	100	100
2nd	90	95
3rd	80	90
4th	70	85
5th	85	85
6th	90	85.83
7th	100	87.86

Wow! Clutch's average falls when his marginal test score is below the average. Then, his average stays constant when his marginal test score equals his average. Finally, his average rises when his marginal is above average!!!! Oh, I can't stand the glee!!!

Here's Clutch graphed (not to scale, but the shape is right):



END OF DIGRESSION. Now, back to costs and Q.

RELATIONSHIP BETWEEN LONG RUN COSTS AND Q

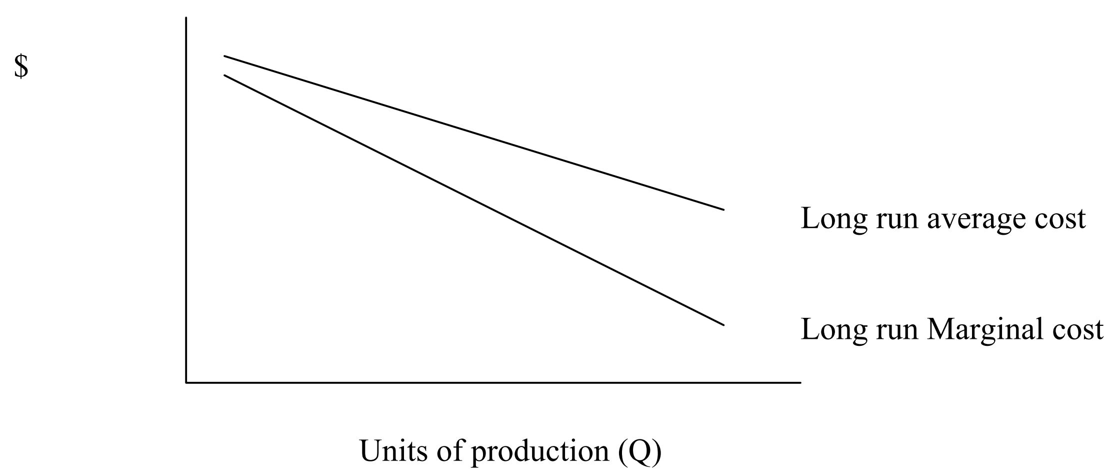
Note 1: Recall that the *long run* is a time period long enough for a firm to vary the use of all of the factors that it employs. (Compare this to the *short run*, a limited time period too short to vary the use of at least one factor.)

Conclusion: All costs are variable in the long run.

Let us look at how marginal and average costs vary with Q. There are many different possibilities, depending upon the technology of the firm.

<u>Possibility 1: Economies of scale:</u> falling long run average costs
For some types of firms, long run average costs (AC) fall as the level of production rises. This is certainly true for products that require mass production, such as automobiles and microchips.

Economies of scale graphed:



<u>Example</u>: A cost function exhibiting economies of scale:

$TC = 10Q^{.5}$

"TC" is total cost

Given the above cost function,

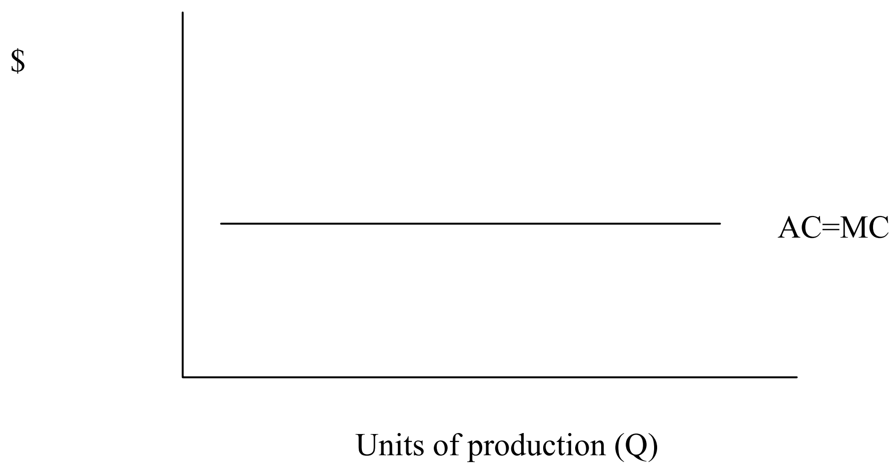
$AC = TC/Q = (10Q^{.5})/Q = 10Q^{-.5}$ ("AC" is long run average cost)

$$MC = \frac{\partial TC}{\partial Q} = 5Q^{-.5}$$ ("MC" means "long run marginal cost'')

Implication of economies of scale: To survive, a firm with this type of production must constantly expand, since the bigger it is, the lower its average costs. If the firm doesn't expand, then surely its competition will, and the larger firm can price its product more cheaply than the smaller firm, eventually driving the smaller firm out of business.

Possibility 2: Absence of Economies of scale: constant long run average costs
For some types of firms, long run average costs (AC) remain constant as the level of production rises. This is usually true for products that require a constant amount of labor per person customer, such as hairstyling and plumbing.

Absence of Economies of scale graphed:



Example: A cost function exhibiting economies of scale:

TC = 10Q

Given the above cost function,

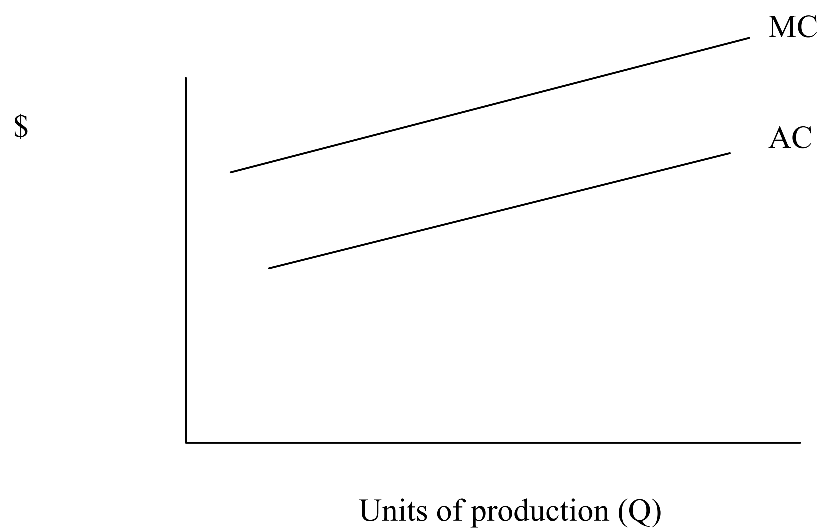
AC = TC/Q = (10Q)/Q = 10

$$MC = \frac{\partial TC}{\partial Q} = 10$$

Implication of absence of economies of scale: Large firms have no cost advantage over smaller firms in this case. Hence one would see smaller firms competing with larger firms in this case.

<u>Possibility 3: Diseconomies of scale:</u> rising long run average costs
It is at least theoretically possible that long run average costs (AC) rise as the level of production rises. In this case, smaller firms would have lower average costs than larger firms, and would drive larger firms out of business. I can't think of any real world examples of this phenomenon.

Disconomies of scale graphed:



<u>Example</u>: A cost function exhibiting diseconomies of scale:

$TC = 10Q^2$

"TC" is total cost

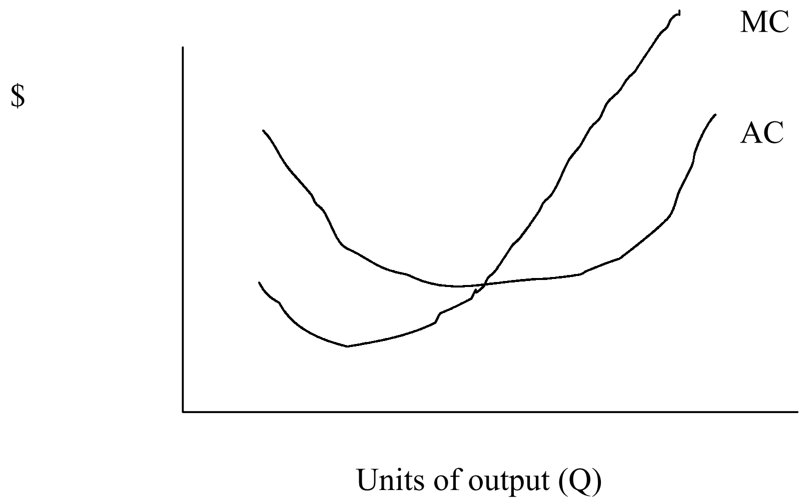
Given the above cost function,

$$AC = TC/Q = (10Q^2)/Q = 10Q$$

$$MC = \frac{\partial TC}{\partial Q} = 20Q$$

<u>Implication of diseconomies of scale</u>: To survive, a firm with this type of production must constantly shrink, since the smaller it is, the lower its average costs. If the firm doesn't shrink, then surely its competition will, and the smaller firm can price its product more cheaply than the larger firm, eventually driving the larger firm out of business. (Not very realistic, is it?)

<u>Possibilities 4 through infinity:</u> Some combination of possibilities 1,2, and 3.
Perhaps most real world firms would be best represented by some combination of possibilities 1, 2, and 3. One popular combination is the "U" shaped AC curve:



<u>Implication of a "U" shaped AC curve</u>: Too small is bad; firms with too low of a production level will have high average costs. But too big is bad; firms that are too big have high average costs (possibly because the larger firm requires more layers of management). A medium size, therefore, provides the cheapest average cost.

We have finished looking at costs in the long run. Let us now turn to short run costs.

RELATIONSHIP BETWEEN SHORT RUN COSTS AND Q

In the short run there are two types of costs—fixed and variable.

FIXED costs do not increase in total as production increases.

Example: For a restaurant, property taxes are a fixed cost; so is the cost to build the restaurant.

VARIABLE costs increase in total as production increases

Example: For a restaurant, food costs and waitstaff costs are variable costs.

Ways to measure costs in the short run:

Total variable costs (TVC): sum of all of the variable costs
Average variable costs (AVC): variable costs per unit of output, TVC/Q
Total fixed costs (TFC): sum of all of the fixed costs
Average fixed costs (AFC): fixed costs per unit of output, TFC/Q
Total costs (TC): sum of all costs, fixed + variable
Average total costs (ATC): total costs evenly spread per unit of output, C/Q
Marginal costs (MC): increase in total cost when Q rises by 1 unit. More formally:

MC = $\frac{\Delta TC}{\Delta Q}$, or using calculus, MC = $\frac{\partial TC}{\partial Q}$

Example of a short run cost function:

$TC = 1000 + 5Q^2$

The "1000" in the above equation is total fixed costs; the "$5Q^2$" is total variable costs.

IT'S INEVITABLE: MARGINAL COSTS MUST EVENTUALLY RISE AS Q RISES IN THE SHORT RUN, FOR ANY AND EVERY FIRM IN THE SHORT RUN.

Why? Because of the Law of Diminishing Returns

Explanation: In the short run, the firm has a fixed factor (say, the land that a farmer can cultivate). The only way to increase production is to use more of the variable factor along with the fixed factor (say, by using more farm workers on the fixed amount of land). It becomes more and more difficult to increase production, as the fixed factor becomes "crowded" with more and more units of the variable factor (e.g. each farm worker has less and less land to work with per person). This makes it more and more COSTLY to increase production; costs increase faster and faster as Q rises. This is the definition of rising marginal cost.

Implication: Recall the relationship between marginal cost and average cost discussed earlier in these notes. *If MC is rising, it must be pulling average variable cost and average total cost up along with it.*

Short run costs and Q: Tabular example

Below is a table representing how the various measurements of short run costs are associated with Q. The numbers in the table are consistent with the inevitable rising MC mentioned above. Be sure that you understand the relationships between total, average, and marginal costs in this table.

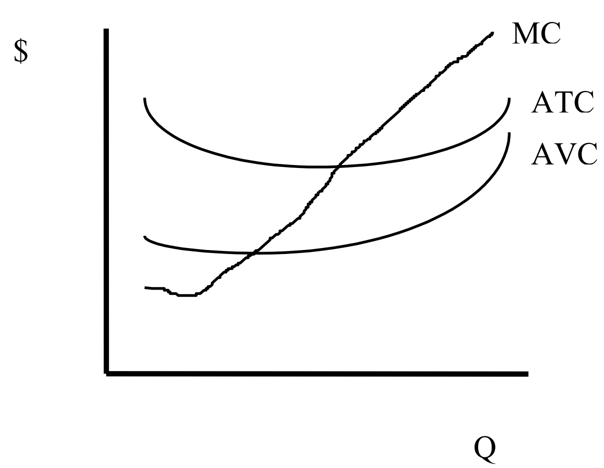
Q	TFC	TVC	TC	MC	AFC	AVC	ATC
0	20	0	20	na	na	na	na
1	20	20	40	20	20	20	40
2	20	30	50	10	10	15	25
3	20	50	70	20	6 2/3	16 2/3	23 1/3
4	20	80	100	30	5	20	25
5	20	120	140	40	4	24	28

Notes about the above table:

1. TC = TFC + TVC
2. ATC = AVC +AFC
3. AFC falls as Q rises. Why? Because fixed costs, TFC, are constant; hence when one divides TFC by Q, one gets a smaller and smaller result as Q rises.
4. MC rises, never to fall again, from Q =2 and on to larger Qs. This is due to the law of diminishing returns-—he concept discussed on the previous page.
5. AVC and ATC eventually begin to rise as Q rises, never to fall again. Why? Because the rising MC is pulling them up.

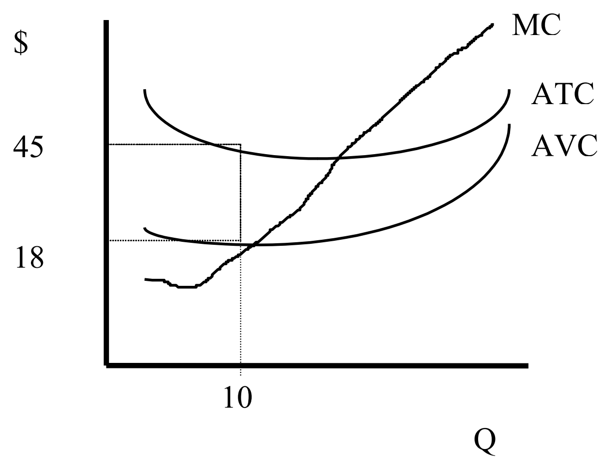
A fairly realistic representation of any firm's short run costs

Any and all firms face rising MC, AVC, and ATC, beyond some level of Q, due to the law of diminishing returns. Hence it is reasonable to represent any and all firms' short run costs with the graph below:



Note on the graph above how ATC, AVC, and MC all eventually rise. Note also that *MC intersects AVC and ATC at the minimum points of both the AVC and ATC curves* (the bottom of each "U").

We can get a lot of cost information from a graph of the type show above, if we add a few numbers to it. Here's an example below



From the graph above, we know:

Q = 10
AVC = $18
ATC = $45

Furthermore, we can calculate

AFC = ATC – AVC = $27

TC = ATC x Q = $450

TFC = AFC x Q = $270

TVC = AVC x Q = $180

My advice, here at the end of these notes:

--Be sure that you truly fully understand all of the different measure of cost. That is, be sure you understand what AC, MC, TC, MC, ATC, AFC, AVC, TVC and TFC are, really.

--Keep in mind the importance of MC. A firm trying to decide to increase production should measure the benefits of expansion against the costs. Well, MC IS THE COST OF EXPANSION. In a future set of notes, mana-profits, we shall devise a measure of the benefits of expansion, and devise a profit-maximizing rule for any firm to follow.