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Macroeconomics

## Study Question: Solow Growth Model

A closed economy with no government is represented as follows:

$y_t = k_t^{.6}$
$n = .02$
$d = .09$

where y is output per worker, k is capital per worker, n is the growth rate of the labor force, and d is the rate of depreciation of the capital stock.

a) Using the above information calculate steady state $y_t$ and $c_t$ assuming a savings rate of 10%

b) Using the above information calculate steady state $y_t$ and $c_t$ assuming a savings rate of 50%

c) Using the above information calculate steady state $y_t$ and $c_t$ assuming a savings rate of 90%

a) First, recall three things:

i) investment per worker = savings per worker
ii) savings per worker = $.10y_t$ (where ".10" is the savings rate)
iii) in the steady state, investment per worker = $(n + d)k_t = .11k_t$

Since savings per worker = investment per worker, substitute what we know:

$$.10y_t = .11k_t$$

Solve the above equation for $y_t$:

$$y_t = 1.1k_t$$

Now recall the production function:

$$y_t = k_t^{.6}$$

From the last two equations we know (by setting them equal):

$$1.1k_t = k_t^{.6}$$

Divide both sides by $k_t^{.6}$:

$$1.1k_t/k_t^{.6} = 1$$

Divide both sides by 1.1:

$$k_t/k_t^{.6} = .90909091$$

Recall that $k_t/k_t^{.6} = k_t^{.4}$ :

$$k_t^{.4} = .90909091$$

Raise both sides to the power of 1/.4

$$(k_t^{.4})^{1/.4} = .90909091^{1/.4}$$

Solve for $k_t$:

$$k_t = .90909091^{2.5}$$

$k_t =$
0.787985611

Now substitute your solution for $k_t$ into the production function:

$$y_t = .787985611^{.6}$$

$y_t =$
0.866784

Since savings is 10% of $y_t$, consumption is the remaining 90%:

$$c_t = .9(.866784)$$

$c_t =$
0.780106

Check your answer: does savings = $(n+d)k_t$ ?

$$.1(.866784) = .11(.787985611)?$$

$$.0866784 = .0866784 \ ! \quad \text{Yes!}$$

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b) First, recall three things:

i) investment per worker = savings per worker
ii) savings per worker = $.50y_t$ (where ".50" is the savings rate)
iii) in the steady state, investment per worker = $(n + d)k_t = .11k_t$

Since savings per worker = investment per worker, substitute what we know:

$$.50y_t = .11k_t$$

Solve the above equation for $y_t$:

$$y_t = .22k_t$$

Now recall the production function:

$$y_t = k_t^{.6}$$

From the last two equations we know (by setting them equal):

$$.22k_t = k_t^{.6}$$

Divide both sides by $k_t^{.6}$:

$$.22k_t/k_t^{.6} = 1$$

Divide both sides by .22:

$$k_t/k_t^{.6} = 4.545454545$$

Recall that $k_t/k_t^{.6} = k_t^{.4}$ :

$$k_t^{.4} = 4.5454545454$$

Raise both sides to the power of 1/.4

$$(k_t^{.4})^{1/.4} = 4.5454545454^{1/.4}$$

Solve for $k_t$:

$$k_t = 4.5454545454^{2.5}$$

$k_t =$
44.04973478

Now substitute your solution for $k_t$ into the production function:

$$y_t = 44.04973478^{.6}$$

$y_t =$
9.690942

Since savings is 50% of $y_t$, consumption is the remaining 50%:

$$c_t = .5(9.690942)$$

$c_t =$
4.845471

Check your answer: does savings = $(n+d)k_t$ ?

$$.5(9.690942) = .11(44.04973487)?$$

$$4.845471 = 4.845471 \quad ! \quad \text{Yes!}$$

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c) First, recall three things:

i) investment per worker = savings per worker
ii) savings per worker = $.90y_t$ (where ".90" is the savings rate)
iii) in the steady state, investment per worker = $(n + d)k_t = .11k_t$

Since savings per worker = investment per worker, substitute what we know:

$$.90y_t = .11k_t$$

Solve the above equation for $y_t$:

$$y_t = .12222222k_t$$

Now recall the production function:

$$y_t = k_t^{.6}$$

From the last two equations we know (by setting them equal):

$$.12222222k_t = k_t^{.6}$$

Divide both sides by $k_t^{.6}$:

$$.12222222k_t/k_t^{.6} = 1$$

Divide both sides by .12222222:

$$k_t/k_t^{.6} = 8.18181818$$

Recall that $k_t/k_t^{.6} = k_t^{.4}$ :

$$k_t^{.4} = 8.18181818$$

Raise both sides to the power of 1/.4

$$(k_t^{.4})^{1/.4} = 8.18181818^{1/.4}$$

Solve for $k_t$:

$$k_t = 8.18181818^{2.5}$$

$k_t$ =
191.4805035

Now substitute your solution for $k_t$ into the production function:

$$y_t = 191.4805035^{.6}$$

$y_t =$
23.40317

Since savings is 90% of $y_t$, consumption is the remaining 10%:

$$c_t = .1(23.40317)$$

$c_t =$
2.340317

Check your answer: does savings = $(n+d)k_t$ ?

$$.9(23.40317) = .11(191.4805035)?$$

$$21.06286 = 21.06286 \ ! \quad \text{Yes!}$$