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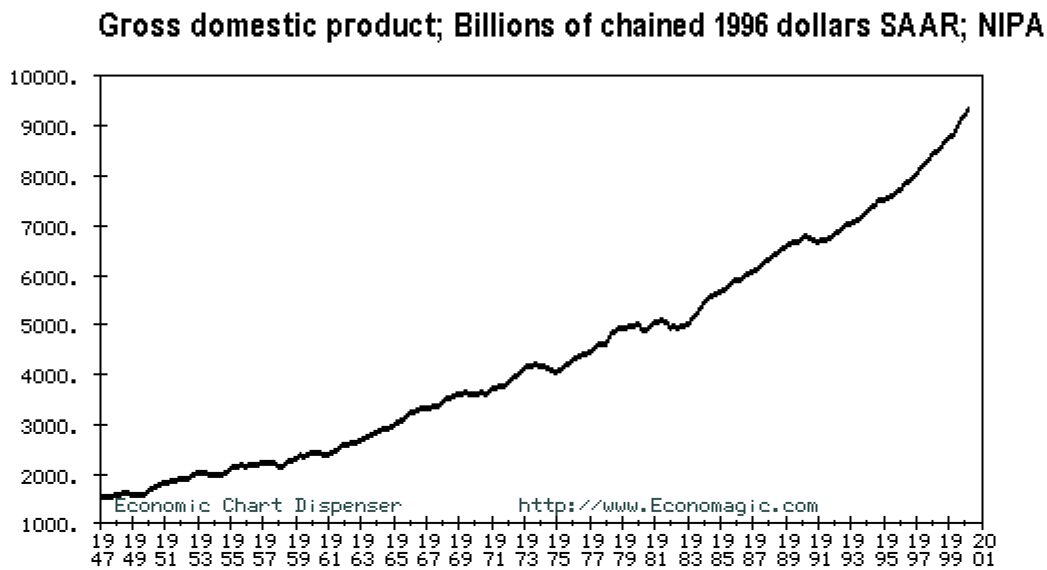
Macroeconomics

Productivity, Output, and Employment

Contents

Introduction

What causes an economy to grow over the long term? Wouldn't it be nice to know? Hey, let's take a look at the growth of real GDP in the U.S. since World War II.



Although growth hasn't been constant, there's a nice upward trend. (Perhaps you can also see that in the 90s, the average growth rate has accelerated—good news for material standards of living.) If we can figure out what causes this nice trend, then perhaps there are things that can be done to keep up the growth or even make it accelerate.

Yes, we are about to begin our journey examining long term economic growth. We want to know why the trend is upward over time. Long term growth theory usually ignores the little dips and bursts in the economy (business cycles), and focuses on the long term trend.

In this chapter, we shall develop a model that looks at three things at the sources of real GDP growth: the amount of physical capital in the economy, the amount of labor in the economy, and the level of technology in the economy (which determines the overall level of factor productivity.) These three things combine in an aggregate production function to determine the GDP of an economy.

After that, we will take a closer look at the aggregate labor market. We want to know what determines the amount of labor working in an economy at any point in time, and we want to know what determines the average wage. We shall develop a model of the aggregate labor market, which looks at labor demand and labor supply and how they interact to determine equilibrium in the labor market.

Now, on with it!

Aggregate Production Function

Think of all of the things that go into the effort of producing things in an economy—all of the people and their individual efforts, all of the raw materials, all of the machinery, and all of the technological know-how. It may seem impossible to incorporate all of these things into a model that lets us simulate the things that determine the size of our economy—the total level of GDP.

Well, it is impossible. But we can simplify a bit, and try to keep (and aggregate) the important things that determine GDP in the model while dismissing the unimportant things. Macroeconomists do this all of the time. What they build are called aggregate production functions.

An aggregate production function shows a precise mathematical relationship between the output of an economy and the things that make that output possible.

Some aggregate production functions that practicing macroeconomists use are quite complex, involving dozens of equations. We shall stick with a simpler version of an aggregate production function, known as a Cobb-Douglas function. Here is the general form of a Cobb-Douglas production function:

$$Y = AK^aN^b$$

What the heck is that? Well, Y represents the gross domestic product, K is the aggregate real value of the physical capital in the economy, N is the number of workers in the economy, a and b are constants, and A is a measure of total factor productivity.

Total factor productivity is a number measuring the overall effectiveness with which capital and labor are used. (The higher, the better—more output per unit of capital and labor.)

Our textbook tells us that this specific form of a Cobb-Douglas production function provides a good representation of how U.S. GDP depends upon capital, labor, and technology:

$$Y = AK^{.3}N^{.7}$$

Using this specific production function, here's a table showing N, K, A, and Y for 1990-1995 in the U.S.: (Y and K are measured in billions of 1992 dollars, and N is millions of workers)

year	**$Y = AK^{.3}N^{.7}$**	**A**	**K**	**N**
1990	6139	16.16	5830	117.9
1991	6079	16.04	5892	116.9
1992	6244	16.34	5979	117.6
1993	6384	16.44	6094	119.3
1994	6609	16.52	6258	123.1
1995	6743	16.52	6473	124.9

Notice:

--the amount of labor is rising over time. This is due mostly to population growth. We shall look at a model of the labor market later in these notes, to see the sort of things that influence the amount of labor in the economy.

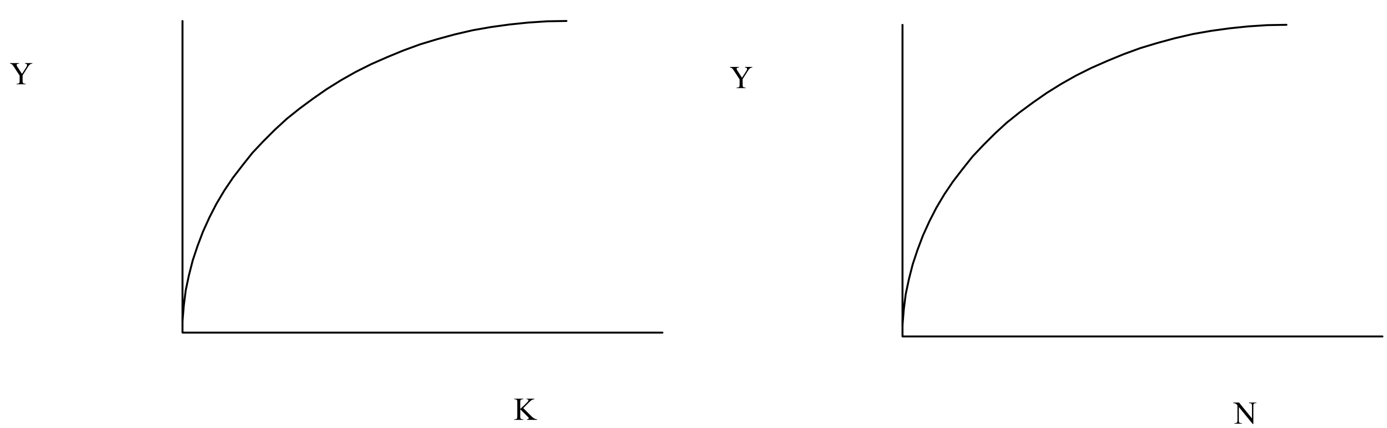
--the amount of capital is rising over time. This is due to Gross Private Domestic Investment, I,--the thing we discussed in the notes file *macro-measure*. The notes file *macro-csi.doc* discusses the determinants of the level of investment in the economy.

--total factor productivity, A, is rising over time. This is due to technological innovations (such as the internet) which make each unit of capital and labor more productive. This is really the key to rising standards of living; when one can get more output out of the same amount of inputs, one can consumer more stuff..

What else can we say about the aggregate production function? Well, let's graph it to get an idea. We'll graph it in two ways

--We'll look at how quickly Y rises as more **K** is employed in the economy.

--We'll look at how quickly Y rises as more **N** is employed in the economy.



What can we learn by looking at the shape of the aggregate production function?

1. Hypothetically speaking, if you add more capital to the economy while keeping labor and technology constant, then output will rise. But each extra unit of capital causes Y to rise by a smaller and smaller amount.

2. Hypothetically speaking, if you add more labor to the economy while keeping capital and technology constant, then output will rise. But each extra unit of labor causes Y to rise by a smaller and smaller amount.

Let's define some important terms that relate to the above two comments—the marginal product of capital and the marginal product of labor:

The Marginal Product of Capital

The marginal product of capital (MPK) measures the increase in Y when K rises by a small amount (say, by 1 unit). In our Cobb-Douglas production function, the marginal product of capital falls as the amount of capital employed rises. This is known as diminishing returns to capital. This means that as you add more and more capital to an economy, output rises but at a slower and slower rate.

There is a formula that we can use to calculate the marginal product of capital. This formula is NOT in the textbook. (For those of you who know calculus, the marginal product of capital is the partial derivative of the production function with respect to K, $\frac{\partial Y}{\partial K}$)

Generally, MPK = $aAK^{a-1}N^{b}$

For our U.S. version of the production function, this means:

MPK = $.3AK^{-.7}N^{.7}$

Let's calculate this value for the U.S. in 1995 at various levels of K:

MPK	K	A	N
1.15538	1000	16.52	124.9
0.71122	2000	16.52	124.9
0.535476	3000	16.52	124.9
0.437807	4000	16.52	124.9

What does this mean? It means, for example, that the employing 1000^{th} unit of capital added about 1.15538 units to total output. Employing the 2000^{th} units added .71122 units to total output. Etc, etc. See how each extra unit of capital is less useful to the producer? This places a limit on how many units of capital that will be employed in the economy, and a limit on the amount of gross private domestic investment. (More on this topic in chapter 4 and in the notes file *macro-cis*, where we'll see that the investor compares the benefit of employing extra capital to its user cost to determine how much capital to buy and employ.)

The Marginal Product of Labor

The marginal product of labor (MPN) measures the increase in Y when N rises by a small amount (say, by 1 unit). In our Cobb-Douglas production function, the marginal product of labor falls as the amount of labor employed rises. This is known as diminishing returns to labor. This means that as you add more and more labor to an economy, output rises but at a slower and slower rate.

There is a formula that we can use to calculate the marginal product of labor. This formula is NOT in the textbook. (For those of you who know calculus, the marginal product of capital is the partial derivative of the production function with respect to N, $\frac{\partial Y}{\partial N}$)

In general, $MPN = bAK^aN^{b-1}$

For our U.S. Cobb-Douglas production function,

$$MPN = .7AK^{.3}N^{-.3}$$

Let's calculate this value for the U.S. in 1995 at various levels of N:

MPN	N	A	K
8.678866	1000	16.52	6473
7.04943	2000	16.52	6473
6.242041	3000	16.52	6473
5.725916	4000	16.52	6473

What does this mean? It means, for example, that the employing 1000^{th} unit of labor would add about 8.678866 units to total output. Employing the 2000^{th} units would add 7.04943 units to total output. Etc, etc. See how each extra unit of labor is less useful to the producer? This places a limit on how many units of labor that will be employed in the economy.

Hey…let's take a closer look at the things that determine the level of employment in the economy (and that determine the average wage). This is important; if we want to forecast growth rates of GDP, then we need to forecast how big the labor supply will be.

We shall examine labor by constructing one huge market for labor in the economy—the (aggregate) labor market!

The Labor Market

Recall from the cobwebs of your mind that a market is an institution or mechanism allowing buyers and sellers to make exchanges. A market is modeled with two groups of participants—buyers, who *demand* the item being bought and sold, and sellers, who *supply* the thing being bought and sold.

In the labor market, labor is being bought and sold. Employers buy, or demand, the labor, and people supply, or sell their labor to employers. We need accurate models of both demand and supply; what things motivate employers to demand labor, and what things motivate people to supply labor? Can we model these things with graphs and equations? You betcha. Let's begin with labor demand.

Labor demand

What's our theory of what motivates employers to employ labor. It ain't 'cuz they're being nice. No—they employ labor because labor produces stuff that the employers can sell to generate revenue.

Here's the idea: Firms hire more labor as long as each additional worker produces stuff of greater value than the wage that the firm must pay the worker. For *example*, if you could hire a worker for $10 that would result in extra production worth $100, then that's a good deal. Indeed, as long as the value of the extra production exceed the wage, then it makes sense to hire more workers.

Let's define some terms to help us formalize this theory:

The marginal revenue product of labor (MRPN) is the value of the production made possible by employing 1 more unit of labor. It equals the marginal product of labor times the product's price.

MRPN = MPN x p

Example: The 10^{th} worker hired allows your production to rise by 5 units. Each units sells for $12. Then:

MRPN = 5 x $12 = $60

You would hire this worker if her wage (w) were less than $60. You would not hire her if her wage were more than $60.

This leads us to the following conclusion:

A firm will continue to hire workers until the point where MRPN = w

From nominal to real variables in labor demand:

Remember how it's not wise to use nominal variables when trying to track changes that occur over time. So let's convert the above boxed condition, which is a nominal condition, into a real condition. First , a digression:

Converting nominal variables into real variables:

Suppose we have data on something measured in dollars over several years, but we want to remove the effects of inflation from our thing. How do we do it? Simple…we divide each nominal value by the value of a price index, then multiply the result by 100.

real value = (nominal value / price index) x 100

This converts all of your dollars into dollars from the base year of the price index.

Example: Here's Nobbynee's salary for the past 3 years, along with the CPI for the past 3 years

year	(nominal) salary	CPI
1998	18,000	98
1999	19,000	100
2000	20,000	104

It's hard to tell how much Nobbynee's purchasing power has changed over the years. Your salary has risen, but so have prices. If we convert Nobbynee's salary from each year to real dollars—a measure of his salary in the dollars of 1999, the base year—then we can compare his real salaries to see what's happened to his purchasing power.

year	(nominal) salary	CPI	real salary
1998	18,000	98	(18,000/98) x 100 = 18367.35
1999	19,000	100	(19,000/100) x 100 = 19000
2000	20,000	104	(20,000/104) x 100 = 19230.77

If you want to compare the purchasing power of Nobbynee's salary from year to year, then using the real salary figures—expressed in constant base year dollars—is a much better thing than using nominal dollars unadjusted for inflation.

End of digression. Now back to our labor demand example.

We want to convert our nominal labor demand condition—MRPN = w—into a real condition. What we need to do is to divide MRPN and w by the average price level, and multiply by 100. But what the heck is the average price level in our model?

Well, here's the thing. We assume that there is only 1 huge labor market, so in effect there's only 1 huge producer hiring the labor. If there's only 1 producer, then there's only 1 product. If there's only 1 product, then there's only 1 price of a product. If so, then the average price level is simply the price of the firm's product, p!!!!!

So let's divide both sides of our labor demand condition by p and divide them by 100:

$$(MRPN/p) \times 100 = (w/p) \times 100$$

Note that the 100s cancel out, so:

$$(MRPN/p) = (w/p)$$

Recall that MRPN = MPN x p. Let's substitute that into our condition:

$$(MPN \times p) / p = (w/p)$$

Note that the p's cancel out in the numerator and denominator on the left side of the equation:

MPN = (w/p)

Let's express this condition in words:

A firm will hire labor until the point where the marginal product of labor equals the real wage.

Let's use our condition in an *example*:

Here's a tabular example:

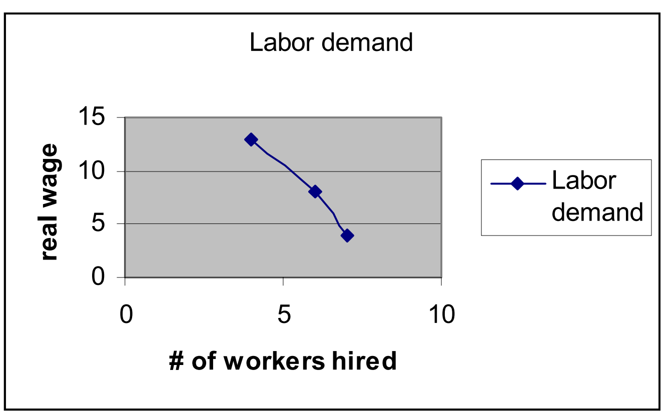
# of workers	marginal product of labor, MPN	real wage	real wage	real wage
1	20	12.99	7.99	3.99
2	18	12.99	7.99	3.99
3	16	12.99	7.99	3.99
4	13	12.99	7.99	3.99
5	10	12.99	7.99	3.99
6	8	12.99	7.99	3.99
7	4	12.99	7.99	3.99
8	1	12.99	7.99	3.99

Suppose the real wage is **12.99**. Then the firm will hire **4** workers, since each of the four workers has a marginal product greater than 12.99.

Now suppose the real wage is **7.99**. Then the firm will hire **6** workers, since each of the four workers has a marginal product greater than 7.99.

Finally, suppose the real wage is **3.99**. Then the firm will hire **7** workers, since each of the four workers has a marginal product greater than 3.99.

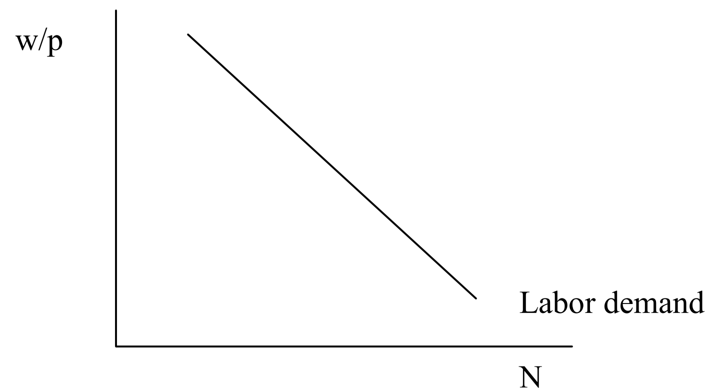
Notice how a lower real wage induces greater employment of labor? If we graphed the three suppositions, each as a data point, and connected the three data points, then we'd have the following:



We have drawn a labor demand curve! Let's generalize:

The labor demand condition, MPN = w/p, implies that the aggregate labor demand curve is downward sloping.

Here's a general drawing of a labor demand curve:



We can also express labor demand as an equation. Here's an example of a linear labor demand equation, where ND stands for labor demand:

ND = 1000 – 10(w/p)

Shifting the ND curve, and changing the ND equation

Remember, the position of the ND curve is based on the condition MPN = (w/p). So if some event changes MPN, the marginal product of labor, then the ND curve will shift.

Example: Suppose new technology makes each worker more productive. Then MPN rises, and the ND curve shifts to the right. The ND equation would also change. In our example above, the ND equation might change

from ND = 1000 – 10(w/p) to ND = 1200 – 10(w/p)

Hey…we're done modeling half the labor market—labor demand. Now let's look at the other half—labor supply.

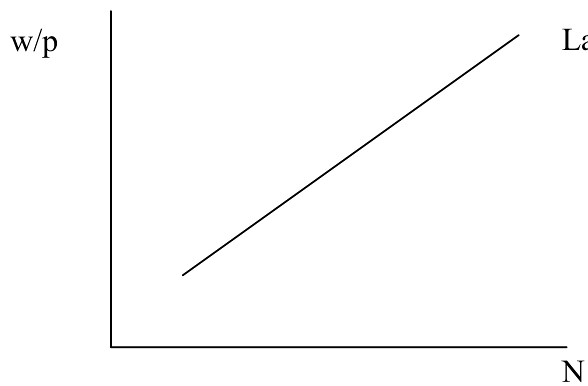
LABOR SUPPLY

What motivates people to supply labor—to work? Well, theory is that each person compares the value of an extra hour of work—the real wage that one would earn—to the value of not working—often called the value of leisure.

It is usually assumed that a higher real wage induces a greater amount of people to work (and those who are already working to work more hours).

A higher real wage induces a greater labor supply

We can use this information to graph a labor supply curve:



We can also create a hypothetical labor supply curve as a straight line with an upward slope. Here's an example, where NS stand for labor supply:

$$NS = 10(w/p)$$

But wait…what else affects labor supply? Wealth is one thing. Suppose that you won the lottery (a massive increase in your wealth); you would be far less likely to work.

Higher wealth reduces labor supply. Lower wealth increases labor supply

Another thing that affects labor supply today is the expected future real wage that you will earn later in life. Why bother to work today, for example, if you expect to earn tons of dough tomorrow?

A higher expected future real wage reduces labor supply. A lower expected future real wage increases labor supply

Another thing that affects labor supply today is a change in the population. A bigger population, for example, usually leads to a bigger labor supply.

Shifting the labor supply curve, and changing the labor supply equation

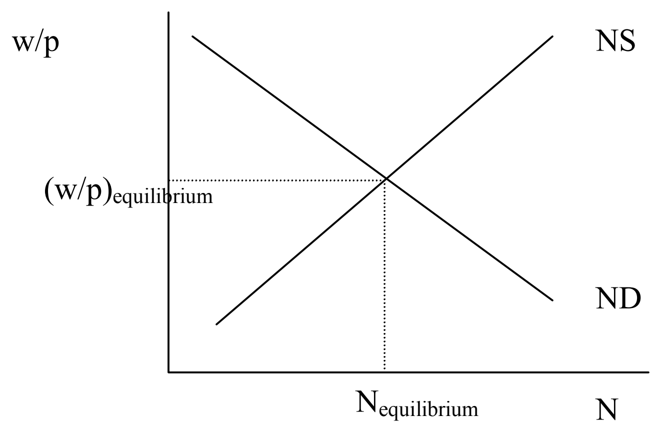
If wealth or the expected future wage or the population changes, then labor supply changes. A reduction in labor supply (caused by higher wealth or a higher expected future real wage or smaller population) would be represented, for example, by a leftward shift of the labor supply curve, or a change in the labor supply equation perhaps

from NS = 10(w/p) to NS = 9(w/p)

Well, we've finished modeling labor supply. Now let's look at what happens in the labor market when buyers and sellers interact: labor market equilibrium.

LABOR MARKET EQUILIBRIUM

So what will the real wage and supply of labor be? It will be at equilibrium—where labor supply equals labor demand. Graphically, it looks like this:



If we have equations for both labor demand and labor supply, then we can calculate equilibrium.

Example: Suppose ND = 100 – 4(w/p) and NS = 6(w/p)

Equilibrium is where ND = NS

So substitute: 100 – 4(w/p) = 6(w/p)

Add 4(w/p) to both sides: 100 = 10(w/p) → (w/p) = 10

So the real wage is 10. Substitute this value into either the ND or NS equations to get the equilibrium amount of labor, N:

N = 100 – 4(10) = 60 or N = 6(10) = 60

EVENTS CHANGING EQUILIBRIUM

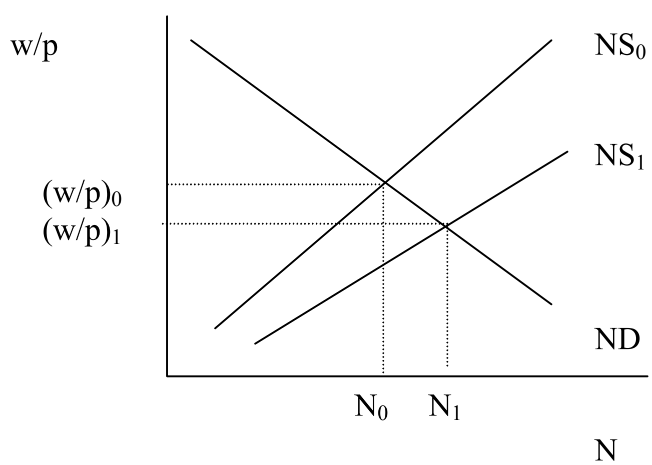
Clearly the labor market doesn't stay in one equilibrium for long; we see the real wage and the size of the labor force change often. What events cause these changes? We know them already; events that change labor demand and/or labor supply cause the equilibrium to change.

This event causes a change in labor demand and hence a change in labor market equilibrium:
- --an event, such as technological change, that affects the marginal product of labor

These events cause a change in labor supply and hence a change in labor market equilibrium:
- --a change in wealth
- --a change in the expected real wage
- --a change in population

<u>A change in equilibrium graphed: example</u>

Suppose that there is an increase in the population. We would expect the real wage to fall and the equilibrium labor force to increase, as follows:



A change in equilibrium using equations: Example

Recall our algebraic example above, with ND = 100 – 4(w/p) and NS = 6(w/p), where we calculated an equilibrium real wage of $10 and N of 60. Well, suppose that *an increase in MPN* causes ND to change to ND = 120 – 4(w/p)

New equilibrium is where ND = NS

So substitute: 120 – 4(w/p) = 6(w/p)

Add 4(w/p) to both sides: 120 = 10(w/p) → (w/p) = 12

So the new real wage is 12. Substitute this value into either the ND or NS equations to get the equilibrium amount of labor, N:

N = 120 – 4(12) = 72 or N = 6(12) = 72

Well, we've laid the groundwork to carefully study production and GDP in an economy. These tools will be useful in future endeavors.