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Financial Markets and Institutions

Credit Markets And Interest Rates

We shall take a closer look at *credit instruments*—the "IOUs" (bonds, mortgages, government securities, and the like) that describe the terms of the repayment of debt. (Credit instruments are also known as *debt instruments*, since one person's credit is always another person's debt.) Remember, these things are bought and sold not just when they are issued, but also on secondary markets. We will see the link between the prices of these credit instruments and their *yield*—the interest rate that they pay if held to maturity. We will also discuss how to keep track of a credit instrument's rate of return per time period (e.g. year) that it is owned. And we'll discuss a thing called *duration*.

There are all kinds of credit instruments with all kinds of repayment terms. Many of these credit instruments can be fit into the following four categories:

Four Types of Credit Instruments

1. Simple Loan:

Provides the borrower with an amount of funds (the *principal*) that must be repaid at the *maturity date*, along with an *interest payment*.

Example: On 1/22/01 Chase Bank loans IBM $1,000,000 that must be repaid at the end of the year along with an interest payment of $50,000.

→maturity date is 1/21/02

→*simple interest rate* is $50,000/$1,000,000 = .05 or 5%

(For a simple 1 year loan, one calculates the simple interest rate, i, as follows: i = interest payment / principal)

2. Fixed Payment Loan

Provides the borrower with an amount of funds that must be repaid in installments. Each installment includes principal and interest payments.

Example: On 1/22/01 Citibank gives you a $120,000 mortgage that must be repaid each month beginning 2/21/01 at $1000 per month for 360 months.

3. Coupon Bond

The borrower (issuer of the bond) pays the bond owner a fixed interest payment (a *coupon* payment) every year until the maturity date, when a specified final amount (the *par* value or *face* value) is also repaid.

Example: On 1/22/01 the U.S. government issues a 10-year Treasury bond with a par value of $10,000 and a coupon of $600.

→maturity date is 1/21/11

→U.S. government will pay 10 coupon payments of $600 beginning 1/21/02, plus will pay an extra $10,000 on 1/21/11
→*coupon rate* is $600/$10,000 = .06 or 6%

4. Discount Bond (a.k.a. zero coupon bond)

This bond is initially bought at a price below its face value, and the face value is repaid at the maturity date. There is no coupon payment.

Example: On 1/22/01 Harris County issues a 5-year bond with a face value of $100,000.

→This bond would be purchased (by a lender) at a price less than $100,000. On 1/21/06 Harris County will pay the owner of the bond $100,000.

With all of these different ways to lend and borrow money, how can prospective borrowers and lenders measure the costs and benefits of each credit instrument, so that each borrower and lender can make the best choice? One thing that one must do is to calculate the *yield to maturity* of each instrument. Before we define this term and discuss how to calculate it, we need to digress a bit and discuss the terms *future value* and *present value*.

Future value

Suppose you are offered a certain bank account—place $100 in the account today, and receive a 5% interest rate (simple interest[1]), for as long as you want.

What will your account's value be in 1 year?

Future value of $100 in 1 year, 5% interest rate = 100 x 1.05 = $105

How about 2 years? (In the second year, the $105 will earn 5% interest.)

Future value of $100 in 2 years, 5% interest rate = 105 x 1.05 = $110.25

How about 3 years? (In the third year, the $110.25 will earn 5% interest.)

Future value of $100 in 3 years, 5% interest rate = 110.25 x 1.05 = $115.7625

Oh, if only there were a formula that could calculate the future value of a sum of money. Hey, there is such a formula! It's on the next page of notes!

$$\textbf{Future value} = \mathbf{PV(1 + i)^n}$$

[1] Throughout our analysis we shall assume simple interest—the equivalent of compounding interest annually. Later, you will encounter other types of interest, e.g. compounded monthly, continuously, etc.

"PV" is present value—the value of the money today
i is the (simple) interest rate
n is the number of years in the future

For example, how much will your $100 investment be worth in 21 years?

Future Value = $100(1 + .05)^{21}$ = $278.5963

(There are more complicated versions of this formula, for investments with uncertain rates of return, and ones with compounding occurring more than once a year, etc. You will encounter some of these more complicated formulas later.)

We have seen how to calculate the future value of $ invested today. But can we calculate the value today—the *present* value—of a future sum? Yes!

Present Value (a.k.a. present discounted value)

Suppose some dude offered you a certain investment—he will certainly give you $1000 ten years from now. How much are you willing to pay him now to receive a certain $1000 in ten years? You will pay less than $1000, since you can invest the money yourself in a Treasury bond that pays 6% rate of return (and let's assume that this is a guaranteed payment).

We can use the future value equation to calculate the maximum amount of money that you would be willing to pay the dude today to get $1000 with certainty in the future:

$FV = PV(1 + i)^n$ (I have used "FV" to abbreviate future value)

Divide both sides by $(1 + i)^n$

$$\frac{FV}{(1+i)^n} = PV$$

Now, for convention's sake, switch sides:

$$PV = \frac{FV}{(1+i)^n}$$ ←Hey! This is the present value equation!!!!

Note that we divide FV by a number greater than 1, that depends on i – the simple interest rate. Hence PV is less than FV. (Hopefully this makes sense to you.) We are said to *discount* the future value; hence i is sometimes called the *discount rate*.

Let's use the present value equation to calculate the most that you would pay the dude today for his investment. Assume that you could receive a 6% interest rate with certainty elsewhere in the economy from other investments.

$$PV = \frac{1000}{(1+.06)^{10}} = \$558.3948$$

The present value of a stream of cash flows

Suppose that you won the lotto, paying you $1,000,000 per year for 25 years. Now some dudette offers to buy your winning lotto ticket for $15 million. Should you accept the offer? (And let's ignore taxes here, as we have done throughout these notes.)

Well, we can use the present value formula 25 times, to calculate the present value of each of the $1,000,000 payments. Then we can add up the 25 results, to get the present value of this stream of twenty-five $1 million cash flows. (We should accept the dudette's $15 million offer if it is greater than the present value of the alternative—the 25 payments.)

A difficulty arises: what do we use for i – the discount rate? We should use the rate of return that we would expect if we accept the dudette's buyout offer and invest the $15 million in some financial instrument. This return is uncertain. Let's just assume an 8% return.

For example:

PV of the first $1 million $PV = \frac{1000}{(1+.08)^{0}} = \$1 \text{ million}$

PV of the second $1 million $PV = \frac{1000}{(1+.08)^{1}} = \$925{,}926$

In the handy table below, I present the PVs of all the 25 payments, at 8% discount rate.

year	**payment**	**pv of payment**
0	1,000,000.00	1,000,000.00
1	1,000,000.00	925,925.93
2	1,000,000.00	857,338.82
3	1,000,000.00	793,832.24
4	1,000,000.00	735,029.85
5	1,000,000.00	680,583.20
6	1,000,000.00	630,169.63
7	1,000,000.00	583,490.40
8	1,000,000.00	540,268.88

9	1,000,000.00	500,248.97
10	1,000,000.00	463,193.49
11	1,000,000.00	428,882.86
12	1,000,000.00	397,113.76
13	1,000,000.00	367,697.92
14	1,000,000.00	340,461.04
15	1,000,000.00	315,241.70
16	1,000,000.00	291,890.47
17	1,000,000.00	270,268.95
18	1,000,000.00	250,249.03
19	1,000,000.00	231,712.06
20	1,000,000.00	214,548.21
21	1,000,000.00	198,655.75
22	1,000,000.00	183,940.51
23	1,000,000.00	170,315.28
24	1,000,000.00	157,699.34
	Sum of PVs	**11,528,758.28**

The dudette's offer of $15 million looks awfully good, compared to the PV of the 25 payments--$11.5 million!

Now that we understand present value, let's discuss this thing called yield to maturity

Yield to Maturity: Loans

Yield to maturity = interest rate that equates today's value of a credit instrument (the price that you'd have to pay to buy the credit instrument today, sometimes known as the *loan value*) with the present value of all the future payments that you'd receive as the owner of the credit instrument.

The yield to maturity (often just called the "yield") represents the interest rate enjoyed by the buyer of the credit instrument (the lender) if she holds it until its maturity date; a higher yield is better for the buyer. (Of course, the buyer must adjust for taxes and risk—topics currently beyond our scope.)

Example 1: Simple Loan

If you loan Zippy $400 today, he'll repay you $490 in one year

Today's price (loan value) = $400
Future payments = $490, 1 year from now

Using the PV formula: $\$400 = 490 / (1 + i)^1$

Yield to maturity = i , so solve for i

(1 + i) = 490/400

1 + i = 1.225

i = .225, or yield to maturity = 22.5%

Example 2. Fixed Payment Loan

You can buy a loan on the secondary market for $1200 that has five yearly payments left of $400 each (with the first of the remaining yearly payments made a year from today).

Today's price (loan value) = $1200
Future payments = $400 1 year from now, $400 2 years from now, $400 3 years from now, $400 4 years from now, and $400 5 years from now

Using the PV formula for a stream of cash flows:

$$1200 = \frac{400}{(1+i)^1} + \frac{400}{(1+i)^2} + \frac{400}{(1+i)^3} + \frac{400}{(1+i)^4} + \frac{400}{(1+i)^5}$$

Yield to maturity = i , so solve for i

Huh? This is a very hard math problem, so it's best to use a financial calculator to solve for i.

To solve for i, we must input into our financial calculator:
1. Today's price (the loan value--1200)
2. The number of payments (5)
3. The amount of each payment—(400)

Then we must compute the yield to maturity

Follow your calculator's instructions.
--> The loan value is usually inputted as a negative number (signifying that it is a cash outflow to the lender), using a button that is perhaps labeled "PV."
--> The number of payments is usually inputted using a button labeled "n."
-->The amount of each payment is usually inputted as a positive number (signifying that it is a cash inflow to the lender) using a button labeled "PMT."
-->One must then usually push a "compute" button, then push a "yield to maturity" button, which may be labeled "I/Y" or "i"

(Also, be sure that your calculator is set so that it is calculating 1 payment per year, and that the payments occur at the end of the year.)

answer (using my Texas Instruments BAII Plus):

yield to maturity = 19.8577%

Example 3: Coupon Bond:

You buy a Treasury bond with a face value of $10,000 for $11,000. It has three coupon payments left, the first of which will be paid one year from now. The coupon rate is 5%.

What is this bond's yield to maturity?

Today's price (loan value) = $11,000
Future coupon payments = $500 1 year from now, $500 2 years from now, and $500 + 3 years from now.
Future repayment of face value = $10,000 3 years from now

Using the PV formula for a stream of cash flows:

$$11{,}000 = \frac{500}{(1+i)^1} + \frac{500}{(1+i)^2} + \frac{500}{(1+i)^3} + \frac{10{,}000}{(1+i)^3}$$

Yield to maturity = i , so solve for i

Huh? This is a very hard math problem, so it's best to use a financial calculator to solve for i.

To solve for i, we must input into our financial calculator:
1. Today's price (the loan value—11,000)
2. The number of payments (3)
3. The amount of each payment—(500)
4. The payment of the par value ($10,000 in 3 years)

Then we must compute the yield to maturity

Follow your calculator's instructions.
--> The loan value is usually inputted as a negative number (signifying that it is a cash outflow to the bond owner), using a button that is perhaps labeled "PV."
--> The number of payments is usually inputted using a button labeled "n."

-->The amount of each payment is usually inputted as a positive number (signifying that it is a cash inflow to the bond holder) using a button labeled "PMT."
-->The repayment of the face value is usually inputted as a positive number (signifying that it is a cash inflow to the bond owner) using a button labeled "FV."
-->One must then usually push a "compute" button, then push a "yield to maturity" button, which may be labeled "I/Y" or "i"

(Also, be sure that your calculator is set so that it is calculating 1 payment per year, and that the payments occur at the end of the year.)

answer (using my Texas Instruments BAII Plus):

yield to maturity = 1.562%

<u>Example 4. Discount Bond</u>

You buy a discount bond (no coupon payment) with a face value of $100,000 for $80,000. The bond matures in exactly 6 years

What is this bond's yield to maturity?

Today's price (loan value) = $80,000
Future coupon payments = none
Future repayment of face value = $100,000 6 years from now

Using the PV formula:

$$80{,}000 = \frac{100{,}000}{(1+i)^6}$$

Yield to maturity = i , so solve for i

This is not a terribly hard math problem, but let's use a financial calculator to solve for i.

To solve for i, we must input into our financial calculator:
1. Today's price (the loan value—80,000)
2. The number of years until the bond is repaid (6)
3. The amount of each payment—(0)
4. The payment of the par value ($100,000)

Then we must compute the yield to maturity

Follow your calculator's instructions.

--> The loan value is usually inputted as a negative number (signifying that it is a cash outflow to the bond owner), using a button that is perhaps labeled "PV."
--> The number of years until the bond is repaid is usually inputted using a button labeled "n."
-->The $0 amount of each payment is usually inputted using a button labeled "PMT."
-->The repayment of the face value is usually inputted as a positive number (signifying that it is a cash inflow to the bond owner) using a button labeled "FV."
-->One must then usually push a "compute" button, then push a "yield to maturity" button, which may be labeled "I/Y" or "i"

(Also, be sure that your calculator is set so that it is calculating 1 payment per year, and that the payments occur at the end of the year.)

answer (using my Texas Instruments BAII Plus):

yield to maturity = 3.789%

Yield to maturity is the best measure of a credit instrument's effective *interest rate*

Relationship Between the Price of a coupon bond and Yield to Maturity (interest rates)

1. If you have to pay *more* than a coupon bond's face value to buy it, then its yield to maturity is *less* than the bond's coupon rate. (Look at example 3 above; the coupon rate is 5% and the yield is only 1.562%)

2. If you have to pay less than a coupon bond's face value to buy it, then its yield to maturity is more than its coupon rate.

3. Implication of 1 and 2 above: If a bond's price rises then its yield falls. If a bond's price falls than its yield rises. (This makes sense if you think of it from the prospective bond buyer's point of view. Remember, the bond buyer is looking for a good investment. When is she more likely to buy a bond with, say, a $1000 coupon—if the bond costs $5,000 or if the bond costs $5,000,000? That's right—when it's cheaper, because the bond is a better investment—provides a higher yield—the cheaper it is to buy.)

Real world implication:
If you hear on the cable channel that "bond *yields* fell today" then you KNOW FOR CERTAIN that bond *prices* rose today.
If you hear on CNNfn that "bond *prices* fell today" then you KNOW FOR CERTAIN that bond *yields* (and the general level of interest rates) rose today.

There is general agreement that yield to maturity is the best measure of the "interest rate" paid by a credit instrument. However, there are other, inferior, ways to measure yields (arising out of history) that are still used today. I shall very briefly discuss two of them below; see the textbook for a more thorough discussion.

Current Yield : The current yield of a coupon bond is the coupon payment (C) divided by the current price of the bond (P)

$$i_c = \frac{C}{P}$$

Two Characteristics of current yield

1. Is better approximation to yield to maturity, nearer price is to par and longer is maturity of bond (see p. 51 of textbook)
2. Change in current yield *always* signals change in same direction as yield to maturity (see p. 52 of text)

Yield on a Discount Basis: Still used today to measure yields on discount bonds (especially Treasury Bills). See p. 53 of text for an explanation of this measure of yield.

Bond Page of the Newspaper

Please bring a recent copy of the Wall Street Journal to class if you wish to follow an in-class discussion of the pages on which bond yields (and other bond information) is displayed.

Distinction Between Real and Nominal Interest Rates

A nominal interest rate (or yield) is the interest rate observed by anyone in the real world; no adjustments are made. All of the yields to maturity that we calculated earlier are nominal yields.

The purchasing power of yields are eroded, however, if there is inflation (a general increase in prices of goods and services). When is a bond with a 10% yield more attractive—when you think that prices will rise by 2% or when you think that they will rise by 20%?

The real yield to a credit instrument = nominal yield – expected inflation rate

Similarly,

Real interest rate = nominal interest rate – expected inflation rate

$$i_r = i - \pi^e$$

1. Real interest rate more accurately reflects true cost of borrowing
2. When real rate is low, greater incentives to borrow and less to lend

if i = 5% and π^e = 0% then:

i_r = 5% - 0% = 5%

if i = 10% and π^e = 20% then

i_r = 10% - 20% = - 10%

Rate of Return

The yield to maturity measures the interest rate of a credit instrument if one holds it until its maturity date. However, many bond holders do not want to hold the thing until it matures; they at least want the option to sell the bond early on secondary markets. (Suppose, for example, that you buy a newly-issued 20-year Treasury bond, thinking that you'll use the repayment of the face value in the year 2021 to pay for your daughter's college. But your in the year 2017 your daughter runs away to join the circus. You may want to sell your bond in 2017.)

But prices of credit instruments change every day (for reasons we'll discuss later in the semester), so there is no guarantee that the price that you can sell your bond tomorrow will equal the price that you paid for it today. (Indeed, only by coincidence will the price of a credit instrument today equal its price on any day in the future.)

The **rate of return** on a credit instrument measures the hypothetical return to the owner of a credit instrument over some time period (e.g. a year), assuming that the owner sells the credit instrument at the end of the time period.

Let's do some examples, then display a formula to calculate rate of return.

Rate of return example 1: You own a discount bond (with no coupon) today that you could sell for $500 today. Exactly 1 year from now you could sell the same bond for $600.

Rate of return = (600 – 500) / 500 = .20 or 20%

Rate of return example 2: You own a coupon bond today that you could sell today for $1000. Exactly 1 year from now it pays a coupon of $100, and you could sell the bond later that day for $1050.

Rate of return = [(1050 – 1000) + 100] / 1000 = .15 or 15 %

Rate of return example 3: You own a coupon bond today that you could sell today for $1500. Exactly 1 year from now it pays a coupon of $100, and you could sell the bond later that day for $1350.

Rate of return = [(1350 – 1500) + 100] / 1000 = -.05 or –5%

Now here's a formula for rate of return (or RET) of a bond:

$$RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$ (see p. 63 of text)

"C" is the coupon payment
"P_t" is the price of the bond now
"P_{t+1}" is the price of the bond at the end of the time period.

Slight digression: Capital gain
When one can sell a financial instrument at the end of a time period for more than its value at the beginning of a time period, one is said to have a *capital gain*. (Similarly, when the price of a financial instrument at the end of a time period is less than its value at the beginning of a time period, one is said to have a *capital loss.)*

Check out the equation for rate of return. The term $\frac{P_{t+1} - P_t}{P_t}$ is the portion of the rate of return attributable to a capital gain. It is known as the *rate of capital gain*.

Who should care about rate of return?
Most owners of credit instruments don't hold them until they mature. Just like stockholders, they'd like to buy low and sell high. So the rate of return on a bond is very important to most bondholders.

Consider a coupon bond that you are considering buying. Among other things, what should you consider when deciding whether or not to buy it?
--How much it costs now.
--The coupon that it pays
--How much you'll be able to sell it for later.

How Rates of Return on bonds fluctuate with interest rates: **interest rate risk**
Recall that when interest rates (yields) rise, bond prices fall. Conclusion: In a time period with rising interest rates, an existing bond will have a capital loss (and a negative rate of capital gain). This will reduce the rate of return on the bond:
→ When interest rates rise, rates of return fall on existing bonds.

Using similar logic:
Recall that when interest rates (yields) fall, bond prices rise. Conclusion: In a time period with falling interest rates, an existing bond will have a capital gain (and a positive rate of capital gain). This will increase the rate of return on the bond:
→ When interest rates fall, rates of return rise on existing bonds.

Interest rate risk is the risk that changing interest rates will change the rate of return on a financial instrument.

Interest rate risk and time to maturity

Note these things:

1. It can be demonstrated mathematically (and you may do so if you wish, using the formula for the present value of a stream of cash flows) that:

→Bonds with longer times to maturity suffer from larger price swings when interest rates change. (See table, p. 63)

Implication: there is more interest rate risk, the longer a bond's time to maturity.

2. If you plan to hold a bond until it matures, then there is (virtually) no interest rate risk, since you receive the face value of the bond at its maturity date.

Duration and Interest-Rate Risk

Duration is a measure of the interest rate risk of a security—the lower the duration, the lower is the interest rate risk. The concept of duration is difficult to explain. Here's the idea:

1. Each remaining bond payment (coupon and/or face value) has a present value.
2. We can assign the present value of each of these payments a weighted maturity—how many years this payment will effectively take to be paid (relative to the other payments). The weight given each payment depends upon
    - i) The size of the present value of the payment, relative to the other payments. (Larger size means larger weight)
    - ii) The number of years in the future that the payment is made. (More years in the future means a larger weight).
3. We add up the weighted maturities of all payments (that we calculated in step 2); this is the bond's duration.

You see, there is more interest rate risk, relatively speaking, if

i) Payments are made far from now. (This is the same point that we made earlier; there is more interest rate risk for bonds with longer times to maturity.)

ii) Large payments are made in the later years relative to the early years. (This is true because interest rates have a higher probability of changing, the longer you wait.)

The formula to calculate duration is given on p.70 of the textbook. It is not a difficult formula, but it is a tedious process. Here are two examples:

Duration Example #1: You own a coupon bond that will pay a coupon of $50 for three more years, beginning one year from now. With the last coupon payment also will come a repayment of the bond's face value, $2,000. The current interest rate is 5%

Step 1: Calculate the present value of each future payment

First coupon payment (1 year from now) PV_a = \$50/(1.05) = \$47.62

Second coupon payment (2 years from now) PV_b = \$50/$(1.05)^2$ = \$45.35

Last coupon payment (3 years from now) PV_c = \$50/$(1.05)^3$ = \$43.19

Payment of face value (3 years from now) PV_d = \$2000/$(1.05)^3$ = \$1727.68

Step 2: Sum all of the present values:

$PV_a+PV_b+PV_c+PV_d$ = \$1863.84

Step 3: Calculate the percentage that each of the present values contributes to the sum that you calculated in step 2

PV_a weight = \$47.62/\$1863.84 = .025549

PV_b weight = \$45.35/\$1863.84 = .024332

PV_c weight = \$43.19/\$1863.84 = .023174

PV_d weight = \$1727.68/\$1863.84 = .926945

Step 4: Multiply each weight that you calculated in step 3 by the number of years that it's in the future:

PV_a weighted duration = .025549 x 1 = .025549

PV_b weighted duration = .024332 x 2 = .048665

PV_c weighted duration = .023174 x 3 = .069521

PV_d weighted duration = .926945 x 3 = 2.780835

Step 5 (the last step): Sum the weighted durations calculated in step 4; the sum is the duration of your bond!

Duration = .025549 + .048665 + .069521 + 2.780835 = **2.92457 years**

Duration example #2: You own a discount (zero coupon) bond that will pay \$2150 three years from now.

(Notice that this bond has the same time to maturity—3 years—as the bond in duration example 1, and it also has the same total payments--$2150—but the payments occur later in the future. So this bond should have more interest rate risk—a larger duration. Let's see! How exciting! I may pee my pants!)

Step 1: Calculate the present value of each future payment. (There's only 1!)

Payment of face value (3 years from now) PV = $\$2150/(1.05)^3 = \$1857.25$

Step 2: Sum all of the present values:

PV = $1857.25

Step 3: Calculate the percentage that each of the present values contributes to the sum that you calculated in step 2

PV weight = $1857.25/$1857.25 = 1

Step 4: Multiply each weight that you calculated in step 3 by the number of years that it's in the future:

PV weighted duration = 1 x 3 = 3

Step 5 (the last step): Sum the weighted durations calculated in step 4; the sum is the duration of your bond!

Duration = 3 = **3 years**

The duration of the bond in example 2—3 years—is greater than the duration of the bond in example 1—2.92457 years. Hence the bond in example 2 has more interest rate risk.